

STA6557 Object Data Analysis SPRING 2017

Days/Time/Room: TT 9:30 AM -10:45 AM OSB 205

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Textbook: Nonparametric Statistic on Manifolds and their Applications, by Vic Patrangenaru, Leif Ellingson, 2015, Chapman Hall/CRC.

Technology: MATLAB, LaTeX.

Prerequisite: STA 5707 or STA5746, STA5334 or STA 5227 or equivalent

Additional Reading: A. Bhattacharya and R. Bhattacharya *Nonparametric Inference on Manifolds With Applications to Shape Spaces*. Institute of Mathematical Statistics Monographs, 2012.

Prerequisite: One of STA 5707, STA 5327, STA 5747, STA 6709 or equivalent.

Course outcomes:After completing this course, the student will be able to:

- determine what is the appropriate sample space for analyzing a given collection of observations of digital images, medical imaging outputs, shapes, unit vectors, functions, etc.
- apply key results in modern statistics, including the central limit theorems or other asymptotic theorems for nonparametric inference on manifolds
- use nonparametric statistical procedures for statistical inference, including large sample theory and nonparametric bootstrap on a smooth sample space

- analyze object data

Course description: Object data analysis is the most general type of data analysis that is known in Statistics. Technically, this is Data analysis on sample spaces with a manifold stratification. Data on manifolds originally arose in astronomy, meteorology, geology, cartography, biology and physics. A drastic increase in the need of data analysis on manifolds occurred with the computer revolution in digital imagery, and the internet. Digital images, which arguably account for the largest types of data available, are today the bread and butter of modern sciences such as Digital Image Analysis, Bioinformatics, Medical Imaging, Computer Vision, Pattern Recognition, Astrophysics, Learning, Earth Sciences, Forensics, etc. Since manifolds are most often nonlinear spaces, fundamental statistical indices of a probability distribution on a manifold, such as mean and covariance have to be redefined in this general context, and their estimation is naturally approached from a nonparametric perspective. The foundation of Nonparametric Statistics on manifolds includes large sample theory on manifolds, density estimation on manifolds, inference on manifolds, and other techniques originating from multivariate analysis. After a recollection of nonparametric multivariate analysis, and a rigorous introduction to abstract manifolds and manifolds as sample spaces, one defines location and spread parameters for distribution on manifolds, and proves the consistency of their sample counterparts. Hypothesis testing and density estimation on manifolds follow. The second part is dedicated to a nonparametric statistical analysis on certain special manifolds arising in statistics, such as axial manifolds, various shape manifolds, Diffusion Tensor Imaging manifolds, learning manifolds, etc. The third part focuses on concrete applications in Astronomy, Image Analysis, Medical Imaging, Bioinfor-

matics, Computer Vision and 3D scene recognition, etc.

Topics covered

1. Data on Manifolds

1.1 Directional and Axial Data.

1.2 Similarity Shape Data and Size and Shape Data

1.3 Digital Camera Imaging Data

1.4 CT scan data

1.5 MRI-DTI data

2. Review of Nonparametric Multivariate Inference.

2.1 Basic Probability Theory.

2.2 Integration on Euclidean Spaces

2.3 Random vectors.

2.4 Parameters and Sampling Distributions of their Estimators

2.5 Consistency and Asymptotic Distributions of Estimators.

2.6 The Multivariate Normal Distribution

2.7 Convergence in Distribution and The Central Limit Theorem.

2.8 Basic Large Sample Theory

2.9 Elementary Nonparametric Inference

2.10 Principal Components Analysis (P.C.A.)

2.11. Multidimensional Scaling

2.12 Nonparametric Bootstrap and the Edgeworth Expansion.

2.13 Nonparametric Function Estimation

3. Basic Geometry of Manifolds 3.1 Manifolds. Submanifolds, Embeddings, Lie

Group actions, Hilbert manifolds

3.2 Riemannian Structures. Curvature. Geodesics.

3.3 Topology of Manifolds

3.3. Basic Algebraic and Differential Topology

3.4. Manifolds in Statistics

4. One sample Inference on Manifolds.

4.1. Fréchet means.

4.2. Consistency of Fréchet sample means

5.2 Asymptotic and bootstrap distributions of Fréchet total variances on manifolds.

5.4 A central limit theorem for Fréchet sample means and bootstrapping.

5.5 The CLT for extrinsic sample means and confidence regions for the extrinsic mean.

5. Two sample tests on Manifolds.

6.1 Introduction

6.2 A two sample test for total extrinsic variances.

6.3 Test for mean change in matched pairs.

6.4 A large sample test for comparing extrinsic means of two independent populations on a manifold.

6.5 The two sample problem on a Riemannian homogeneous space.

6.6 Nonparametric Bootstrap for Two Sample Tests.

7. Function estimation on Manifolds

7.1 Function estimation on Homogeneous spaces and Lie groups

7.2 Statistical Inverse Estimation

Applications of Nonparametric Statistics on Manifolds

- 15.1 Diffusion Tensor Imaging Brain Analysis.
- 15.2. Tests for Equality of Generalized Frobenius Means via Cholesky Decompositions
- 16. Applications to Directional Data Analysis.
 - 16.1 The Pluto controversy
 - 16.2 The solar nebula theory
 - 16.3 Why Pluto is not a major planet of the Solar System.
- 17 Applications of Direct Similarity Shape Analysis.
 - 17.1. University School X-ray data analysis
 - 17.3 Louisiana Experimental Glaucoma Study data analysis
- 18 Similarity Shape Analysis.
 - 18.2 Planar direct similarity shape manifolds
 - 18.3 Reflection shape manifolds in higher dimensions.
 - 18.5 Extrinsic mean planar shapes and their estimators
 - 18.6 Asymptotic distribution of mean shapes
- 19. Direct Similarity Shape Analysis of Planar Contours.
- 20 Estimation of Mean Skull Size and Shape from CT scans, with Applications in Planning Reconstructive Plastic Surgery in Young Adults.
- 21. A statistical Analysis of 2D Electrophoretic Gels based on Affine Shapes 358
- 23. Applications to 3D machine vision.
 - 23.1. Projective Shape Analysis of 3D Scenes from Pinhole Camera Images.
 - 23.3. Projective Shape and 3D Reconstruction
 - 23.4. Nonparametric Estimation and Testing for the Projective Shape of a 3D Config-

uration

23.5. Face recognition example.

24. Two Sample Tests for Mean Projective Shapes and Scene Recognition

25 Mean Glaucomatous Projective Shape Change Detection From Stereo Pair Images.

26. Computational Issues.

Grading: The course grade will be calculated on the basis of a midterm report (40%), and a final presentation(60%). The project is meant to be created as a draft of a paper to be submitted on ArXiv (or equivalent).

University Attendance Policy: Excused absences include documented illness, deaths in the family and other documented crises, call to active military duty or jury duty, religious holy days, and official University activities. These absences will be accommodated in a way that does not arbitrarily penalize students who have a valid excuse. Consideration will also be given to students whose dependent children experience serious illness.

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Americans With Disabilities Act: Students with disabilities needing academic ac-

accommodation should: (1) register with and provide documentation to the Student Disability Resource Center; and (2) bring a letter to the instructor indicating the need for accommodation and what type. This should be done during the first week of class.

This syllabus and other class materials are available in alternative format upon request.

For more information about services available to FSU students with disabilities, contact the: Student Disability Resource Center

874 Traditions Way

108 Student Services Building

Florida State University

Tallahassee, FL 32306-4167

(850) 644-9566 (voice)

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sdrc@admin.fsu.edu

<http://www.disabilitycenter.fsu.edu/>

Disclaimer: This syllabus provides a general plan; deviations may be necessary.