Nonparametric Bayesian Statistics

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Alternatives to DP: Pitman-Yor processes

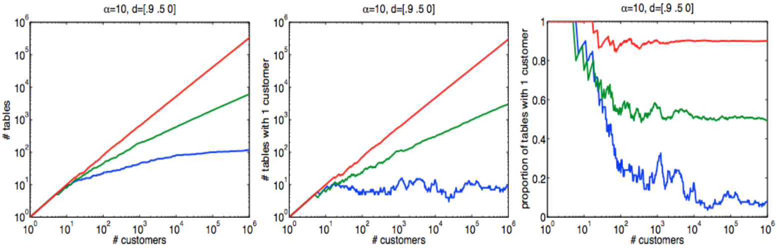
- Pitman-Yor Processes are also known as Two-parameter Poisson-Dirichlet Processes.
- Chinese restaurant representation:

$$P(n^{\text{th}} \text{ customer sit at table } k, 1 \leq k \leq K) = \frac{n_k - d}{n-1+\alpha}$$
$$P(n^{\text{th}} \text{ customer sit at new table}) = \frac{\alpha + dK}{i-1+\alpha}$$

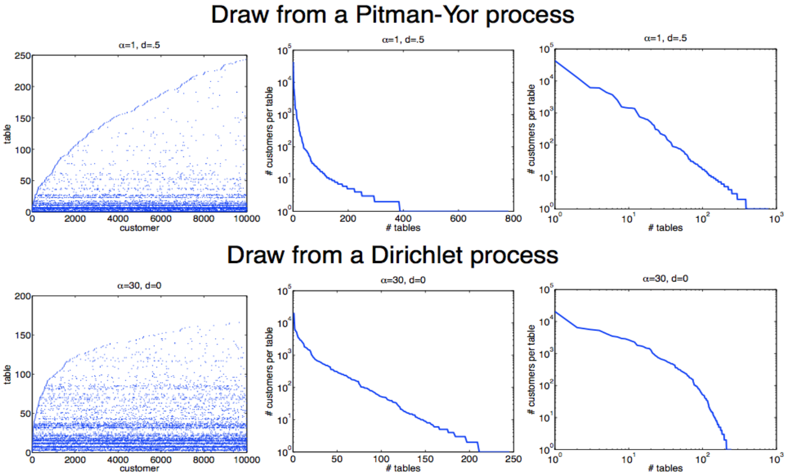
  where $0 \leq d < 1$ and $\alpha > -d$.
- When $d = 0$ the Pitman-Yor process reduces to the DP.

Pitman-Yor processes

- Two salient features of the Pitman-Yor process:
  - With more occupied tables, the chance of even more tables becomes higher.
  - Tables with smaller occupancy numbers tend to have lower chance of getting new customers.
- The above means that Pitman-Yor processes produce Zipf's Law type behaviour.



Pitman-Yor processes



Stick-breaking contraction for PY processes

- We can relax the priors on β_k in the stick-breaking construction:

$$G = \sum_{k=1}^{\infty} \pi_k \delta_{\theta_k^*} \qquad \pi_k = \beta_k \prod_{l=1}^{k-1} (1 - \beta_l)$$

$$\theta_k^* \sim H \qquad \beta_k \sim \text{Beta}(a_k, b_k)$$

- We get the DP if $a_k = 1$, $b_k = \alpha$.
- We get the Pitman-Yor process if $a_k = 1 - d$, $b_k = \alpha + kd$.
- To ensure that $\sum_{k=1}^{\infty} \pi_k = 1$, we need β_k to not go to 0 too quickly:

$$\sum_{k=1}^{\infty} \pi_k = 1 \quad \text{almost surely iff} \quad \sum_{k=1}^{\infty} \log(1 + a_k / b_k) = \infty$$

Two group DDP

- One possibility for a two group DDP is to let

$$
\begin{aligned}
f(y \mid x) &= \sum_{h=1}^{\infty} \pi_h \mathrm{N}(y; \mu_{xh}, \tau^{-1}) \\
\mu_{1h} &\sim \pi \delta_{\mu_{0h}} + (1-\pi)\delta_{\beta_h}, \\
\mu_{0h} &\sim N(\mu, \sigma_\mu^2), \beta_h \sim N(\mu_\beta, \sigma_\beta^2),
\end{aligned}
$$

- One can test for equality of distributions.