STA 4442/5440 Midterm 1 Review Sheet

This guide contains a list of some important concepts/formulas and corresponding textbook problems (bold, in parentheses).

This guide is **NOT** intended to be used as your only study resource; rather, it should help you navigate your notes, textbook, and homework assignments as you study for your first exam.

Chapter 1: Counting

Chapter 2 & 3: Axioms of Probability, Conditional probability and Independence

- Terminology: experiment, sample space S, elementary outcome e, event (E)
- $P(A)$: probability of an event A
  1. $0 \leq P(A) \leq 1$
  2. $P(S) = 1$
  3. $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ for mutually exclusive events A_i.
- Methods of assigning probability
  1. Uniform model: all outcomes equally likely,
  $$P(A) = \frac{\text{number of outcomes in } A}{\text{number of outcomes in } S}$$
  2. Alternative (long-run relative frequency) model: perform experiment many times, set
  $$P(A) = \text{rel. freq. of } A \text{ in } N \text{ trials} = \frac{\text{number of times } A \text{ occurs in } N \text{ trials}}{N}$$
- Event relations: complement (A^c), union ($A \cup B$), intersection ($A \cap B$ or $A \cap B$); Venn diagram
- Law of complement: $P(A) = 1 - P(A^c)$
- Addition law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Incompatible/mutually exclusive events: $P(A \cap B) = 0$
- Finding probabilities by using a frequency table
- Conditional probability of A given B:
  $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
  Equivalent form: multiplication law $P(A \cap B) = P(B)P(A|B)$
- Law of Total probability: $P(A) = P(A \mid B)P(B) + P(A \mid B^c)P(B^c)$.
- Bayes Theorem: $P(A \mid B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|A^c)P(A^c)}$
- Independence - 3 ways to check:
  1. $P(A \cap B) = P(A)P(B)$
  2. $P(A|B) = P(A)$
  3. $P(B|A) = P(B)$
  4. $P(B \mid A) = P(B \mid A^c)$

Chapter 4: Random variables

- A random variable is a real valued function defined on the sample space S, $X : S \to \Re$. A discrete random variable can take countably many values $x_1, x_2, \ldots$.
- For a discrete random variable X, the probability mass function (p.m.f.) is defined as $p(x) = P(X = x)$ and the cumulative distribution function (c.d.f) is defined as $F(x) = P(X \leq x)$.
- For a discrete random variable X, $E(X) = \sum_{i=1}^{\infty} x_i p(x_i)$, $E(g(X)) = \sum_{i=1}^{\infty} g(x_i)p(x_i)$ and $V(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2$.