Conditional probability - Class 5

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1 Conditional probability

1. A small business with 120 workers and 2 employee benefits, A and B. (Venn box diagram)

	A	A^c	
B	64	34	98
B^c	7	15	22
	71	49	120

   Calculate the prob. that an employee chosen at random has the retirement benefit (A) given that she takes the health-care benefit (B). Restrict our attention to the rst row: 64/98

2. If the sample space S is finite, and all outcomes are equally likely, it is often convenient to compute $P(E \mid F)$ by using F as the sample space as

$$P(E \mid F) = |E \cap F|/|F|$$

   Count $|E \cap F|$ in the reduced sample space F. (Note that $E \cap F$ is an event in F since $E \cap F \subset F$.)

3. An urn contains 10 white balls, 5 yellow balls, and 10 black balls. Choose a ball at random. Suppose we know that it is not black. What is the prob. that it is yellow? $P(Y \mid B^c) = P(Y \cap B^c)/P(B^c) = (5/25)/(15/25)$ Alternative: Consider the reduced sample space B^c or $Y \cup W$. Then $P(Y \mid B^c) =$ P(choose a yellow out of white and yellow) = 5/15 = 1/3

4. The multiplication rule for conditional probability

$$P(E \cap F) = P(E \mid F)P(F)$$
$$P(E_1 \cap E_2 \cap E_3 \ldots \cap E_n) = P(E_1)P(E_2 \mid E_1)P(E_3 \mid E_1 \cap E_2)$$

5. A useful formula for calculating probabilities (Total probability formula)

$$P(E) = P(E \mid F)P(F) + P(E \mid F^c)P(F^c)$$

In general if $\cup_{i=1}^{n} F_i = S$, then

$$P(E) = \sum_{i=1}^{n} P(E \mid F_i) P(F_i)$$