Conditional probability - Class 7

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1 Conditional Probability

1. Property: If E does not depend on the occurrence or non occurrence of F, then E and F are independent.
   Proof: If $P(E \mid F) = P(E \mid F^c)$, then $P(E) = P(E \mid F)P(F) + P(E \mid F^c)P(F^c) = P(E \mid F)$.

1. In answering a multiple-choice question, a student either knows the answer or guesses with probability 1/2 each. Assume P(he answers the question correctly given he guesses) = 1/5 (there are 5 multiple choice alternatives) and P(he answers the question correctly given he knows the answer) = 1 .

   What is the prob that he knew the answer given that he answered the question correctly?

   $P(C \mid K^c) = 1/5, P(K) = P(K^c) = 1/2$. And of course $P(C \mid K) = 1$. What is $P(K \mid C)$? $P(K \mid C) = 0.5 \times 1/(0.5 \times 1 + 0.5 \times 0.2) = 5/6$

2. Stores A, B, C have 50, 75, and 100 employees and, respectively, 50, 60, and 70 percent of them are women. Resignations are equally likely among all employees regardless of sex. One employee resigns, and this is a woman. What is the prob that she works in Store C? (We know $P(A), P(B), P(C), P(w|A), P(w \mid B), P(w \mid C)$ Use Bayes formula to calculate $P(C \mid w)$)

3. 2 dice are rolled - what is the probability that a sum of 5 occurs before a sum of 7. (E: event that a 5 occurs before a 7, F: first trial results in a 5, G results in 7, H is neither 5 nor 7) $P(E) = P(E \mid F)P(F) + P(E \mid G)P(G) + P(E \mid H)P(H)$ and $P(E \mid F) = 1, P(E \mid G) = 0, P(E \mid H) = P(E)$. Also $P(F) = 4/36, P(G) = 6/36, P(H) = 26/36$. So $P(E) = 4/36 + 26/36P(E)$ implying $P(E) = 2/5$.

4. There are n types of coupons, and each new one collected is independently of type i with probability p_i, $\sum_{i=1}^{n} p_i = 1$. Suppose k coupons are to be collected. If A_i is the event that there is at least one type i coupon among those collected, then for $i \neq j$, find (a) $P(A_i)$; (b) $P(A_i \cup A_j)$; (c)$P(A_i \mid A_j)$

   $P(A_i) = 1 - P(A_i^c) = 1 - (1 - p_i)^k, P(A_i \cup A_j) = 1 - P(A_i^c \cap A_j^c) = 1 - (1 - p_i - p_j)^k$

5. The color of a persons eyes is determined by a single pair of genes. A newborn child independently receives one eye gene from each of its parent and the gene it receives

from a parent is equally likely to be either of the two eye genes of that parent. B: brown eye gene, b: blue eye gene, gene B is dominant while gene b is recessive

(a) bb (genotype): the person will have blue eyes

(b) BB : the person will have brown eyes

(c) Bb (or bB) : the person will have brown eyes

(a) Suppose both of Mr. Smith's parents have brown eyes and Mr Smith's sister has blue eyes, what is the probability that Mr. Smith has brown eyes? (3/4)

(b) Suppose Smith and both of his parents have brown eyes and sister has blue eyes. (What does this imply?) What is P(Smith possesses a blue-eyed gene)? Because Smith's sister has blue eyes, her genotype is bb. Consequently, both of Smith's parents (who are brown-eyed) must have genotypes Bb, since the sister must inherit a b gene from each parent. However, Smith himself could have genotype BB or Bb. Let E_1 and E_2 be the events that Smith has genotype BB or Bb, respectively, and let F be the event that Smith has brown eyes. Then the probability that Smith has genotype Bb is

$$P(E_2 \mid F) = \frac{P(F \mid E_2)P(E_2)}{P(F)} = \frac{1.1/2}{3/4} = 2/3.$$