1 Example

Read Example 5d from the Sheldon Ross book (8th edition)

2 Bose-Einstein statistics

Problem numbers 31 and 32 (in Homework 1) from the Sheldon and Ross book (8th edition) are direct applications of Bose Einstein Statistics - see also Section 1.6 of Sheldon and Ross book.

In quantum statistics, Bose-Einstein statistics (or more colloquially BE statistics) is one of two possible ways in which a collection of indistinguishable particles may occupy a set of available discrete energy states. The aggregation of particles in the same state, which is characteristic of particles obeying Bose-Einstein statistics, accounts for the cohesive streaming of laser light and the frictionless creeping of superfluid helium. The theory of this behaviour was developed (1924-25) by Albert Einstein and Satyendra Nath Bose, who recognized that a collection of identical and indistinguishable particles can be distributed in this way.

A much simpler way to think of Bose-Einstein distribution function is to consider that n particles are denoted by identical balls and r shells are marked by $r-1$ line partitions. It is clear that the permutations of these n balls and $r-1$ partitions will give different ways of arranging bosons in different energy levels. Total number of ways is $\binom{n+r-1}{r-1}$.

2.1 Reformulation of BE statistics

A reformulation is to consider the number of ways n indistinguishable balls distributed in r distinguishable urns. This is same as the number of non-negative integer-valued vectors $(x_1, x_2, \ldots, x_n)$ such that

$$x_1 + x_2 + \ldots + x_n = n$$

To that end let us compute the number of positive integral solutions of the above equation. You want a partition of $1, 2, \ldots, n$. Imagine n objects with $n-1$ spaces in between and you want to put $(r-1)$ items in those $n-1$ spaces. Clearly, there are $\binom{n-1}{r-1}$ ways. To obtain the number of non-negative solutions, note that the number of is same as the positive integral solutions of

$$y_1 + \ldots + y_r = n + r$$

(seen by letting $y_i = x_i + 1, i = 1, \ldots, r$).

Then from the preceding discussion by replacing n by $(n+r)$, the number of non-negative integer-valued vectors $(x_1, x_2, \ldots, x_n)$ such that $x_1 + x_2 + \ldots + x_n = n$ is precisely

$$\binom{n-r+1}{r-1}$$