Random Variables - Class 16

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1 Expectation and variance

$E(X) = \int_{-\infty}^{\infty} xf(x)dx$ $V(X) = E(X - E(X))^2$, $E(aX + b) = aE(X) + b, V(aX + b) = a^2V(X)$

1. The density function of X is given by

$$f(x) = \begin{cases} 2x, 0 \leq x \leq 1 \\ 0, otherwise \end{cases}$$

Find $E(X)$. ($E(X) = \int_{\infty}^{\infty} xf(x)dx = 2/3$)

2. For a non-negative continuous random variable Y (which means $P(Y < 0) = 0$ or $f_Y(t) = 0$ for $t \leq 0$)

$$E(Y) = \int_0^{\infty} P(Y > y)dy \tag{1}$$

3. Example: A stick of length 1 is split at a point U that is uniformly distributed over (0, 1). Determine the expected length of the piece that contains the point a, $0 \leq a \leq 1$? Claim: greater than 1/2. Given U, let $L(U)$ denote the length of the substick that contains the point a. So

$$L(U) = \begin{cases} 1 - U, U < a \\ U, U \geq a \end{cases}$$

Hence

$$\begin{aligned} E(L(U)) &= \int_0^a (1-u)du + \int_a^1 udu \\ &= 1/2 + a(1-a) \end{aligned}$$

2 Important rule

Leibnitz rule:

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(t)dt = g(f_1(x))f_1'(x) - g(f_2(x))f_2'(x)$$

2.1 Problem

Suppose that if you are s minutes early for an appointment, then you incur the cost cs, and if you are s minutes late, then you incur the cost ks. Suppose that the travel time from where you presently are to the location of your appointment is a continuous random variable having probability density function f. Determine the time at which you should depart if you want to minimize your expected cost.

Observe that

$$C_t(X) = \begin{cases} c(t - X), if X \leq t \\ s(X - t), if X > t. \end{cases}$$

Differentiate $E(C_t(X))$ with respect to t and equate to 0 to get the solution.