

## STA 4442/5440 Final Exam

December 14, 2012

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

**Pledge:** I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

**Signature:** \_\_\_\_\_

### INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 2 hrs (10:00 A.M to 12:00 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 8 problems and is worth 120 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.	Prob. No.	Max Points	Earned Pts.
1	10		6	10	
2	15		7	15	
3	15		8	20	
4	15				
5	20				

**TOTAL:** \_\_\_\_\_



**Question 1.** (10 pts.) Roll three fair dice independently. Find the conditional probability that there are at least two sixes given that there is at least one six.

**Question 2.** (15 pts.) While watching a game of Champions League football in a cafe, you observe a person supporting Manchester United. What is the probability that the person was actually born within 25 miles of Manchester ?

Assume that

- the probability that a randomly selected person in a typical local bar environment is born within 25 miles of Manchester is  $1/20$ .
- the probability that a person supports Manchester United given that he was born within 25 miles of Manchester is  $7/10$ .
- the probability that a person supports Manchester United given that he was NOT born within 25 miles of Manchester is  $1/10$ .

(Hint: Use Bayes' Theorem)

**Question 3.** (15 pts.) Jill sends her resume to 1000 companies she finds on monster.com. Each company responds with probability  $3/1000$  (independently of what all the other companies do). Let  $R$  be the number of companies that respond.

a) Compute  $E[R]$ .

b) Compute  $Var[R]$ .

c) Use a Poisson random variable approximation to estimate the probability  $P(R = 3)$ .

**Question 4.** (15 pts.)  $X$  and  $Y$  are two discrete random variables taking values  $-1, 0$  and  $+1$  each with joint probability given by

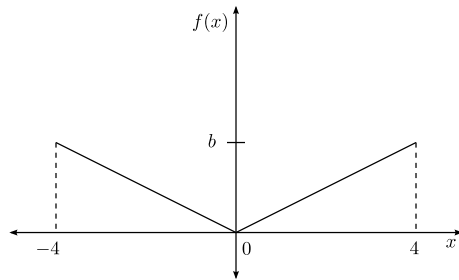
Table 1: Joint probability Table

$Y \downarrow X \rightarrow$	-1	0	+1	Total
-1	0	1/4	0	1/4
0	1/4	0	1/4	1/2
+1	0	1/4	0	1/4
Total	1/4	1/2	1/4	1

a) Find  $\text{Cov}(X, Y)$ .

b) Find whether  $X$  and  $Y$  are independent or not.

**Question 5.** (20 pts.) The figure is the probability density curve of the random variable  $X$ .



- Find  $b$  so that  $f(x)$  is a probability density function.
- What is  $P(-4 \leq X \leq 3)$ ?
- What is  $P(X = 1)$ ?
- What is  $E(X)$ ?

**Question 6.** (10 pts) Nine percent of men are color blind. Researchers need at least 50 men with this trait, so they randomly select 600 men. Estimate the probability that at least 50 color blind men are in the sample (Use normal approximation to binomial and continuity correction).



**Question 7.** (15 pts) A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?

**Question 8.** (20 pts) Each day, a newsboy buys newspapers from the publisher for  $c_1$  cents each, sells them for  $c_2$  cents each, and recycles the unsold papers (if any) getting  $c_3$  cents for each. Note that  $c_2 > c_1 > c_3$ . Let  $H$  denote the number of papers that the newsboy purchases each day. The demand for papers is a discrete random variable  $X$  that takes on nonnegative integer values. Do NOT assume that  $X$  is a binomial random variable. Let  $F(u)$  denote the CDF of  $X$ .

- a) Express the probability that the newsboy is able to sell all  $H$  papers in terms of  $F(u)$ .
- b) One day, the newsboy decides to buy one additional paper in the hope of selling it and increasing his profit. Express the probability that he is unable to sell the additional paper in terms of  $F(u)$ . Be sure you understand the difference between “not being able to sell the  $(H + 1)$ -th paper” and “being able to sell all  $H$  papers but not the extra  $(H + 1)$ -th paper.”
- c) Find  $A(H + 1)$ , the average additional profit from the sale of the extra (that is,  $(H + 1)$ -th) paper.