

## STA 4442/5440 Final Exam Practice

December 11, 2013

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

**Pledge:** I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

**Signature:** \_\_\_\_\_

### INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 2 hrs (12:30 A.M to 2:30 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 8 problems and is worth 120 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.	Prob. No.	Max Points	Earned Pts.
1	10		6	20	
2	10		7	20	
3	10		8	15	
4	20				
5	15				

**TOTAL:** \_\_\_\_\_



**Question 1.** (10 pts.) An IRS agent receives a batch of 15 tax returns that were flagged by computer for possible tax evasions. Suppose, unknown to the agent, 3 of these returns have illegal deductions and the other 12 are in good standing. If the agent randomly selects 4 of these returns for audit, what is the probability that:

- a) All three of the returns that have illegal deductions are selected?
- b) At least 2 have illegal deductions?

**Question 2.** (10 pts.) On a Monday afternoon, 167 customers will be observed during check-out and the number paying by card will be recorded. Records from the store suggest that 58% of customers pay by card. Approximate the following probabilities using the normal approximation to the binomial. Remember to use continuity correction.

- a) Fewer than 101 will pay by card.
- b) Between 88 and 100 (inclusive) will pay by card.

**Question 3.** (10 pts.) Four equally qualified undergraduate students apply for jobs at the university bookstore.

Student	Sex	Year
1	M	Junior
2	M	Junior
3	F	Sophomore
4	M	Sophomore

Two students will be chosen at random to receive jobs. Let

$A$  = [Selected students are of the same sex]

$B$  = [Selected students are of the same year]

- Find the probability of  $A \cup B$ . (*Hint: List all possible outcomes. Then draw a Venn diagram.*)
- Are  $A$  and  $B$  independent? Explain why or why not.
- Find the probability of  $\bar{A} \cap \bar{B}$ .

**Question 4.** (20 pts) A fair die is tossed and its outcome is denoted by  $X$ . After that  $X$  independent fair coins are tossed and the number of heads obtained is denoted by  $Y$ . Compute

a)  $P(Y = 4)$

b)  $P(X = 5 \mid Y = 4)$

c)  $E(Y)$

d)  $E(XY)$

**Question 5.** (15 pts) There are 10 balls in an urn numbered 1 through 10. You randomly select 3 of those balls. Let the random variable  $Y$  denote the maximum of the three numbers on the extracted balls. Find the probability mass function of  $Y$ . You should simplify your answer to a fraction that does not involve binomial coefficients. Then calculate  $P[Y \geq 7]$ .

**Question 6.** (20 pts) The bid that a competitor makes on a real estate property is estimated to be somewhere between 0 and 3 million dollars. Specifically, the bid  $X$  is viewed to be a continuous random variable with density function.

$$f(x) = \begin{cases} c(9 - x^2), & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

You make a bid (without knowing the competitors bid). The higher of the two bids win.

- a) Find the value of  $c$  that makes  $f(x)$  a legitimate density function?
- b) Find the cumulative distribution function,  $F(x)$ . Use the cumulative distribution to determine the probability that you lose the bid if you make a bid of 2 million? 1 million?
- c) Find the expected value and standard deviation for the competitors bid. What is the probability that the competitors bid is within one standard deviation of the mean?
- d) How much should you bid so that you have a 90% chance of winning?



**Question 7.** (20 pts) Which of the following statements are always true? Give brief reasons.

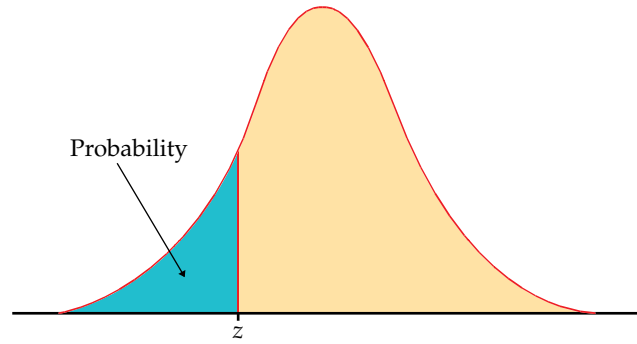
a)  $E(XY) = E(X)E(Y)$

b)  $E(X + Y) = E(X) + E(Y)$

c)  $E\left(\frac{X}{Y}\right) = \frac{E(X)}{E(Y)}$

**Question 8.** (15 pts.) The life of a certain variety of insects (measured in hours) is normally distributed with mean  $\mu$  and variance  $\sigma^2$ . Suppose 95% of the insects lives exceeding 2 hours and 5% of them lives exceeding 20 hours. Find the mean ( $\mu$ ) and the standard deviation ( $\sigma$ ) of the normal distribution. (Given  $\Phi(1.645) = 0.95$ , where  $\Phi$  denotes the standard normal c.d.f)

Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

**TABLE A**

## Standard normal probabilities

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

