STA 4442/5440 Final Exam Review Sheet

This guide contains a list of important concepts/formulas and links to material to be studied for the exam. This guide is **NOT** intended to be used as your only study resource; rather, it should help you navigate your notes, textbook, exams, and homework assignments as you study for your final.

Chapter 1: Review of combinatorics

Chapter 2 & 3: Axioms of Probability, Conditional probability, Independence

- Terminology: experiment, sample space S, elementary outcome e, event,
- $P(A)$: probability of an event A
  1. $0 \leq P(A) \leq 1$
  2. $P(A) = \sum_{all\, e\, in\, A} P(e)$
  3. $P(S) = \sum_{all\, e\, in\, S} P(e) = 1$
- Methods of assigning probability
  1. equally likely,
  $$P(A) = \frac{\text{number of outcomes in } A}{\text{number of outcomes in } S}$$
  2. Alternative (long-run relative frequency) model: perform experiment many times, set
  $$P(A) = \text{rel. freq. of } A \text{ in } N \text{ trials} = \frac{\text{number of times } A \text{ occurs in } N \text{ trials}}{N}$$
- Event relations: complement (A^c), union ($A \cup B$), intersection ($A \cap B$ or $A \cap B$); Venn diagram
- Law of complement: $P(A) = 1 - P(A^c)$
- Addition law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Incompatible/mutually exclusive events: $P(A \cap B) = 0$
- Conditional probability of A given B
  $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
  Equivalent form: multiplication law $P(A \cap B) = P(B)P(A|B)$
- The Bayes' Theorem
  $$P(A|B) = \frac{P(B \mid A)P(A)}{P(B)}$$
- Independence - 3 ways to check:
  1. $P(AB) = P(A)P(B)$
  2. $P(A|B) = P(A)$
  3. $P(B|A) = P(B)$
- Law of total probability $P(A) = P(A \mid B)P(B) + P(A \mid B^c)P(B^c)$

Chapter 4: Discrete Random Variables

- Terminology: random variable, probability distribution
- Discrete vs. continuous random variables
- Listing distinct values of a discrete random variable X
- Discrete probability distribution $p(x_i) = P(X = x_i)$
  1. $p(x_i) \geq 0$ for each value x_i of X
  2. $\sum_{i=1}^{k} p(x_i) = 1$
- Expectation, variance, and standard deviation σ of a discrete probability distribution
- Bernoulli trials
- Binomial
- Poisson
- Geometric
- Binomial distribution for large n and very small p can be approximated by a Poisson distribution with parameter $\lambda = np$.

Chapter 5: Continuous distributions

- Terminology: continuous random variable X, probability density curve, probability density function (pdf)
- Properties of a pdf:
  1. Total area under probability density curve is 1
  2. $P(a \leq X \leq b)$ = area under probability density curve between a and b
  3. $f(x) \geq 0$ for all x
- Meaning of a pdf:
  1. $f(x) \neq P(X = x)$
  2. $P(X = x) = 0$
  3. Only meaningful to talk about probability that a continuous r. v. X lies in an interval
  4. $P(a \leq X \leq b)$ = (area to the left of b) - (area to the left of a)
  5. Expectation, variance and standard deviation of a continuous random variable
- Population $100p$-th percentile: area of p to the left, $1 - p$ to the right
  - Lower (first) quartile = 25th percentile
  - Second quartile (median) = 50th percentile
  - Upper (third) quartile = 75th percentile
- Standardized variable $Z = \dfrac{X - \mu}{\sigma}$ has mean 0, standard deviation 1

- Normal distribution $N(\mu, \sigma)$
  - Symmetric, bell-shaped curve
  - μ locates the center; for this distribution, μ is also the median
  - σ describes the spread: lower values $\rightarrow$ less spread (more concentration about the mean)
- Standard normal distribution $N(0, 1)$
  - Mean $\mu = 0$
  - Standard deviation $\sigma = 1$
  - Table gives $P(Z \leq z) =$ area under curve to the left of z
- Properties of the standard normal distribution
  1. $P(Z \leq 0) = .5$
  2. $P(Z \leq -z) = 1 - P(Z \leq z) = P(Z \geq z)$
- Find an area given an interval of z values
- Find a z value given an area under the curve
- Probability calculations with normal distributions: standardize, then use standard normal table
- Normal approximation to the binomial, using continuity correction
- Uniform distribution, Exponential distribution

Chapter 6: Jointly distributed random variables

- Joint pmf $p(x, y), \sum_{\text{all x,all y}} p(x, y) = 1$
- $X \sim Bin(n, p)$, $Y \sim Bin(m, p)$, then $X + Y \sim Bin(m + n, p)$ if (X, Y) are independent
- $X \sim Pois(\lambda)$, $Y \sim Pois(\mu)$, then $X + Y \sim Poisson(\lambda + \mu)$ if (X, Y) are independent
- $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, then $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ if (X, Y) are independent
- $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, then $X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$ if (X, Y) are independent
- Conditional probability of a discrete random variable X given another discrete random variable $Y = y$ is $p(x, y)/p_Y(y)$.
- $X \sim Poiss(\lambda), Y \sim Poiss(\mu)$, (X, Y) are independent, $X \mid (X + Y) = k \sim Bin(k, \frac{\lambda}{\lambda+\mu})$.
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- $X \sim Geom(p), Y \sim Geom(p)$, (X, Y) are independent, $P(X = i \mid (X + Y) = k) = 1/(k - 1)$.

Chapter 7: Properties of Expectation

- $E(X+Y) = E(X) + E(Y)$
- $Cov(X,Y) = E(XY) - E(X)E(Y)$
- $Var(X+Y) = Var(X) + Var(Y)$ if X, Y are independent
- $Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, $-1 \leq Corr(X,Y) \leq 1$
- $E(X \mid Y) = \sum_{allx} xp(x \mid y)$
- $E(E(X \mid Y)) = E(X)$