Warm-up problems from STA 5326 **Name:** ....

STA 5327 (Instructor: Debdeep Pati) **FSUID:** ....

Work your problems in the space provided. Show all work clearly. Justify your answers. Draw a box around your final answer.

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1. Suppose X and Y are independent exponentially distributed random variables with parameter $\beta > 0$ (and pdf $f(x) = \beta^{-1}\exp(-x/\beta)$ for $x > 0$). Consider transformations $V = X + Y$ and $W = X/(X+Y)$.

(a) Find the joint distribution of V and W.

(b) Specify the marginal distributions of V and W. Are V and W independent?

2. Consider $U_1, \ldots, U_{300} \sim$ i.i.d. uniform$(-1, 1)$. Use the central limit theorem to find an approximation of the distribution of $\sum_{i=1}^{300} U_i/10$.

3. Suppose that $X|Y = p$ has a Binomial(n, p) distribution for $0 < p < 1$ and that Y has a U$(0, 1)$ distribution. Find $E(X)$ and $var(X)$.

4. Let $X_1, X_2, \ldots \overset{\text{i.i.d.}}{\sim}$ expo(1) (with pdf $f(x) = e^{-x}1_{(0,\infty)}(x)$). For every $n \geq 1$, Define $Z_n = \max\{X_1, \ldots, X_n\}/\log n$.
(a) Find the pdf of Z_n and $E(Z_n)$.

(b) Find $\lim_{n\to\infty} P(Z_n \leq 1)$. Does $Z_n \overset{d}{\to} 0$?

5. (X, Y) has a joint pdf

$$f(x,y) = \begin{cases} Cxy & \text{if } 0 < y < 1; 0 < x < y \\ 0 & \text{o.w.} \end{cases}$$

(a) Find the conditional pdf of $X \mid Y = y$ for $0 < y < 1$.

(b) Find the conditional pdf of $Y \mid X = x$ for $0 < x < 1$.

(c) Find $E(X \mid Y = y)$ for $0 < y < 1$.

6. Let X, Y be independent random variables distributed as geometric(p). Set $Z = X + Y$. Find the conditional distribution of $X \mid Z = z$. Verify that $E(X) = E(E(X \mid Z))$.

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7. Let $(X, Y) \sim N_2(0, 0, 1, 1, \rho)$. Define $V = (X - \rho Y)/\sqrt{1 - \rho^2}, W = Y$. Find the joint distribution of V and W.

8. Let X_n be a sequence of random variables with $X_n \sim U(1 - 1/n, 1)$. Show that $X_n \xrightarrow{P} 1$.

9. Let $X_1, \ldots, X_{10}$ be i.i.d. $N(0, 1)$. Find $E[(X_1 - X_2 + X_3 - X_4 + \ldots + X_9 - X_{10})^2]$.