6.30 X: Jill's score Y: Jack's score, $X \sim N(170, 20^2)$, $Y \sim N(160, 15$

(a) $P(Y > X) = P(Y - X > 0)$ $\overset{Y-X}{\cancel{X-Y}} \sim N(-10, 15^2+20^2)$

$$= P\left(\frac{(Y-X)-(-10)}{\sqrt{15^2+20^2}} > \frac{0-(-10)}{\sqrt{15^2+20^2}}\right)$$

$$= P(Z > 0.4)$$

$$= 1 - P(Z \le 0.4)$$

$$= \boxed{0.3446}$$

(b) $P(X+Y > 350) = P\left(\frac{X+Y-330}{\sqrt{15^2+20^2}} > \frac{380-330}{\sqrt{15^2+20^2}}\right)$ $X+Y \sim N(330, 15^2+20^2)$

$$= P(Z > 0.8)$$

$$= 1 - P(Z \le 0.8)$$

$$= \boxed{0.2119}$$

6.32 $\lambda = \tau X = 0.2 \times 10 = 2$ X: # of typos $X \sim \text{poisson}(2)$

(a) $P(X=0) = \frac{e^{-\lambda}\lambda^i}{i!} = \frac{e^{-2}\lambda^0}{0!} = \boxed{e^{-2}}$

(b) $P(X \geq 2) = 1 - P(X=1) - P(X=0)$

$$= 1 - \frac{e^{-2}\cdot(2)^1}{1!} - e^{-2}$$

$$= \boxed{1 - 3e^{-2}}$$

6.34 This is the sum of independent geometric random variables problem

$p_1 = 0.3$ $q_1 = 0.7$, $p_2 = 0.4$, $q_2 = 0.6$

$$P(\zeta_2 = k) = p_2 q_2^{k-1} \frac{p_1}{p_1 - p_2} + p_1 q_1^{k-1} \frac{p_2}{p_2 - p_1}$$

$$P(\zeta_2 > 12) = P(\zeta_2 = 13) + P(\zeta_2 = 14) + \cdots$$

$$= \underline{p_2 q_2^{13-1} \frac{p_1}{p_1 - p_2}} + \frac{p_1 q_1^{13-1}}{1} \cdot \frac{p_2}{p_2 - p_1} + \underline{\frac{p_2 q_2^{14-1}}{1} \cdot \frac{p_1}{p_1 - p_2}} + p_1 q_1^{14-1} \cdot \frac{p_2}{p_2 - p_1} + \cdots$$

common multiple for "__"

common multiple for "~"

$$= \frac{p_2 q_2^{12} \frac{p_1}{p_1 - p_2} (1 - q_2^N)}{1 - q_2} + \frac{p_1 q_1^{12} \frac{p_2}{p_2 - p_1} (1 - q_1^N)}{1 - q_1}, \quad (N \to \infty)$$

$$= \frac{\cancel{p_2} q_2^{12}}{\cancel{1 - q_2}} \cdot \frac{p_1}{p_1 - p_2} + \frac{\cancel{p_1} q_1^{12}}{\cancel{1 - q_1}} \cdot \frac{p_2}{p_2 - p_1}$$

$$= 0.6^{12} \cdot \frac{0.3}{0.3 - 0.4} + \frac{0.7^{12}}{1} \cdot \frac{0.4}{0.4 - 0.3}$$

$$= \boxed{4 \cdot 0.7^{12} - 3 \cdot 0.6^{12}}$$

6.38

(a) $P(X = j, Y = i) = \frac{1}{5} \cdot \frac{1}{j}$, $j = 1, \ldots, 5$, $i = 1, \ldots, j$

choose X from $\{1, \ldots, 5\}$ choose Y from $\{1, \ldots, X\}$

(b) $$P(X = j \mid Y = i) = \frac{P(X = j, Y = i)}{P(Y = i)}$$

$P(Y=5) = \frac{1}{5} \cdot \frac{1}{5}$

$P(Y=4) = \frac{1}{5 \cdot 5} + \frac{1}{4} \cdot \frac{1}{5}$

$P(Y=3) = \frac{1}{5 \cdot 5} + \frac{1}{4 \cdot 5} + \frac{1}{3 \cdot 5}$

$P(Y=2) = \frac{1}{5 \cdot 5} + \frac{1}{4 \cdot 5} + \frac{1}{3 \cdot 5} + \frac{1}{2 \cdot 5}$

$P(Y=1) = \frac{1}{5 \cdot 5} + \frac{1}{4 \cdot 5} + \frac{1}{3 \cdot 5} + \frac{1}{2 \cdot 5} + \frac{1}{1 \cdot 5}$

so $P(Y = i) = \sum_{k=i}^{5} \frac{1}{5k}$

$$P(X = j \mid Y = i) = \frac{\frac{1}{5j}}{\sum_{k=i}^{5} \frac{1}{5k}}$$

6.38 (c) $P(X=j) = \frac{1}{5}$

$$P(X=j)\cdot P(Y=i) = \frac{1}{5}\cdot\sum_{k=i}^{5}\frac{1}{5k} \neq P(X=j, Y=i)$$

So X and Y are $\boxed{\text{not independent}}$.

6.40.

(a) $P(Y=1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$P(Y=2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$$P(X=1 \mid Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)}$$

$$= \frac{P(1,1)}{P(Y=1)}$$

$$= \frac{\frac{1}{8}}{\frac{1}{4}}$$

$$= \boxed{\frac{1}{2}}$$

$$P(X=1 \mid Y=2) = \frac{P(1,2)}{P(Y=2)}$$

$$= \frac{1/4}{3/4}$$

$$= \boxed{\frac{1}{3}}$$

$$P(X=2 \mid Y=1) = \frac{P(2,1)}{P(Y=1)} = \frac{1/8}{1/4} = \boxed{\frac{1}{2}}$$

$$P(X=2 \mid Y=2) = \frac{P(2,2)}{P(Y=2)} = \frac{1/2}{3/4} = \boxed{\frac{2}{3}}$$

(b) $P(X=1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$, $P(X=2) = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$.

$P(1,1) \neq \frac{1}{4}\cdot\frac{3}{8} = P(Y=1)\cdot P(X=1)$, similarly, $P(X=j, Y=i) \neq P(X=j)\cdot P(Y=i)$

So X and Y are $\boxed{\text{not independent}}$.

6.40 (c)

$X \in \{1, 2\}$

$Y \in \{1, 2\}$

$XY \in \{1, 2, 4\}$

(1,1) (1,2) (2,2)

(2,1)

$X+Y \in \{2, 3, 4\}$

(1,1) (1,2) (2,2)

(2,1)

$X/Y \in \{1, \frac{1}{2}, 2\}$

(1,1) (1,2) (2,1)

(2,2)

$$P(XY \le 3) = 1 - P(XY > 3)$$

$$= 1 - P(XY = 4)$$

$$= 1 - P(2,2)$$

$$= 1 - \frac{1}{2}$$

$$= \boxed{\frac{1}{2}}$$

$$P(X+Y > 2) = 1 - P(X+Y \le 2)$$

$$= 1 - P(X+Y = 2)$$

$$= 1 - P(1,1)$$

$$= 1 - \frac{1}{8}$$

$$= \boxed{\frac{7}{8}}$$

$$P(X/Y > 1) = 1 - P(X/Y \le 1)$$

$$= 1 - P(X/Y = 1) - P(X/Y = \tfrac{1}{2})$$

$$= 1 - P(1,1) - P(1,2) - P(1,2)$$

$$= P(2, \cancel{2}\, 1)$$

$$= \boxed{1/8}$$