STA 4442/5440 HW1 Solutions

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No. 10

(a) $8! = \boxed{40{,}320}$

(b) $2 \times 7! = \boxed{10{,}080}$

(c) $4! \cdot 4! \times 2 = \boxed{1152}$

(d) $5! \cdot 4! = \boxed{2{,}880}$

(e) $4! \cdot 2^4 = \boxed{384}$

No. 13

$$\binom{20}{2} = \frac{20!}{2!\,18!} = \boxed{190}$$

No. 19

(a) Two Solutions

One:

$$\underbrace{\binom{8}{3}\binom{4}{3}}_{\text{Do not contain each of the two men}} + \underbrace{\binom{8}{3}\binom{2}{1}\binom{4}{2}}_{\text{Contain exactly one of them}} = \boxed{896}$$

The other solution:

$$\underbrace{\binom{8}{3}\binom{6}{3}}_{\text{Contain all the combinations}} - \underbrace{\binom{8}{3}\binom{4}{1}}_{\text{Contain two of the men refuse to serve together}} = 896$$

(b) Two solutions

one: $\binom{6}{3}\binom{6}{3} + \binom{2}{1}\binom{6}{2}\binom{6}{3} = \boxed{1{,}000}$

the other: $\binom{8}{3}\binom{6}{3} - \binom{6}{1}\binom{6}{3} = 1{,}000$

(c) $\underbrace{\binom{7}{3}\binom{5}{3}}_{\text{Neither feuding party serves}}+\underbrace{\binom{7}{2}\binom{5}{3}}_{\text{feuding woman serves}}+\underbrace{\binom{7}{3}\binom{5}{2}}_{\text{feuding man serves}}=\boxed{910}$

No. 31

(a) Number of nonnegative integer solutions of $x_1+x_2+x_3+x_4=8$.
Hence, answer is $\binom{11}{3}=\boxed{165}$

(b) here it is the number of positive solutions — hence answer is $\binom{7}{3}=\boxed{35}$.

No. 32

(a) Number of nonnegative solution of $x_1+\cdots+x_6=8$. Answer $\binom{13}{5}=\boxed{1,287}$.

(b) (Number of solutions of $x_1+\cdots+x_6=5$) X (Number of solutions of $x_1+\cdots+x_6=3$)
$=\binom{10}{5}\binom{8}{5}=252\times 56=\boxed{14,112}$

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No. 8

There are $\binom{n+m}{r}$ groups of size r. As there are $\binom{n}{i}\binom{m}{r-i}$ groups of size r that consist of i men and r-i women, we see that

$$\binom{n+m}{r}=\sum_{i=0}^{r}\binom{n}{i}\binom{m}{r-i}$$

No. 9.

$$\binom{2n}{n}=\sum_{i=0}^{r}\binom{n}{i}\binom{n}{n-i}=\sum_{i=0}^{r}\binom{n}{i}^2 \xlongequal{\text{change notation } i \text{ to } k} \sum_{k=0}^{n}\binom{n}{k}^2$$

No. 11.

The number of subsets of size k that have i as their highest number member is equal to $\binom{i-1}{k-1}$, the number of ways of choosing k-1 of the numbers $1,\cdots,i-1$. Summing over i yields the number of subsets of size k.

"Points on the plane" Problem

1. $\binom{10}{2} = \boxed{45}$ (choose 2 points from 10 points)

2. We can choose a directed path of lenth two uniquely by ~~using~~ choosing the starting point, the middle point, and the end point.

$10 \cdot 9 \cdot 8 = \boxed{720}$

3. $\binom{10}{3} = \boxed{120}$

Each triangle corresponds to six directed paths of length two — we can choose which two edges and the direction.

4. From (1), we know there are totally 45 segments.

$\binom{45}{4} = \boxed{148,995}$

5. The number of ways to choose four segments that include a triangle is

$\binom{10}{3} \times 42 = 5,040$

We pick ~~up~~ three vertices that will form the triangle, then choose any one of the remaining segments. The probability is

$\frac{5,040}{148,995} \approx \boxed{3.38\%}$