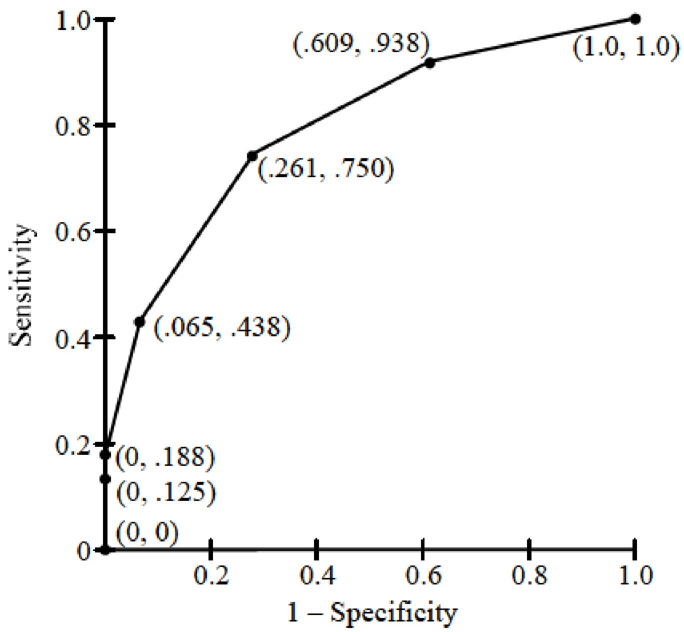
3.83 The sensitivity $= \frac{12}{16} = .75$.

3.84 The specificity $= \frac{34}{46} = .739$.

3.85 We have the following table of sensitivities and specificities according to the cutoff point used

Cutoff point for dementia	sensitivity	specificity
< 0	0	1.0
≤ 5	.125	1.0
≤ 10	.188	1.0
≤ 15	.438	.935
≤ 20	.750	.739
≤ 25	.938	.391
≤ 30	1.0	0

3.86 The ROC curve is a plot of sensitivity vs 1-specificity for different cutoff values. This is shown below.



3.87 From the table in the solution to Problem 3.85 or the ROC curve in the solution to Problem 3.86, the only cutoff value that achieves these criteria is ≤ 20.

3.88

$$\text{The area} = \frac{1}{2}\left[(.188+.438)(.065)+(.438+.750)(.261-.065)+(.750+.938)(.609-.261)+(.938+1.0)\times(1-\right.$$

$$= \frac{1}{2}(1.619) = .809$$

It means that for a randomly selected pair of demented and non-demented individuals, there is an 81% probability that the demented individual will have a lower score than the non-demented individual.

3.100 The probability = $.5 \times .5 = .25$.

3.101 The probability = $.25 \times .25 = .0625$.

3.102 We use Bayes' Theorem. Let C = (dominant with complete penetrance) and A = {2 out of 2 offspring affected}. We wish to compute $Pr(C|A)$ and are given that $Pr(C) = \frac{1}{2}$. Thus, from Bayes' Theorem:

$$Pr(C|A) = \frac{Pr(A|C)Pr(C)}{Pr(A|C)Pr(C) + Pr\left(A|\overline{C}\right)Pr\left(\overline{C}\right)} = \frac{.25(.50)}{.25(.50)+.0625(.50)} = \frac{.25}{.3125} = .80.$$

The probability of .80 is a posterior probability.

3.103 We let D = {3rd child is affected}. From the total probability rule,

$$Pr(D|A) = Pr(D|C) \times Pr(C|A) + Pr\left(D|\overline{C}\right) \times Pr\left(\overline{C}|A\right) = .50(.80)+.25(.20) = .45.$$