Hw2 for 6448 (due March 3) (Debdeep Pati)

1. Suppose we want to sample from $\mathrm{N}_p(\mu, \Sigma)$ where $\mu = \Sigma\Phi^{\mathrm{T}}\alpha$ and $\Sigma = (\Phi^{\mathrm{T}}\Phi + D^{-1})^{-1}$ where $D \in \mathbb{R}^{p\times p}$ a symmetric positive definite matrix, $\Phi \in \mathbb{R}^{n\times p}$ and $\alpha \in \mathbb{R}^{n\times 1}$. Prove that the following algorithm generates a sample $\theta \sim \mathrm{N}_p(\mu, \Sigma)$.
   (a) Sample $u \sim \mathrm{N}(0, D)$ and $\delta \sim \mathrm{N}(0, I_n)$.
   (b) Set $v = \Phi u + \delta$.
   (c) Solve $(\Phi D \Phi^{\mathrm{T}} + I_n)w = (\alpha - v)$.
   (d) Set $\theta = u + D\Phi^{\mathrm{T}}w$.
2. Generate n observations from

$$0.5\,\mathrm{N}(0,1) + 0.5\exp(\text{mean} = 2).$$

   Then $EX_1 = \mu_0 = 1$. Test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$ at 5% using significance level using
   (a) Standard normal approximation.
   (b) Empirical likelihood.

   Try $n = 50, 200$. Find 0.05% confidence intervals of μ in all the cases using i) Bootstrap and ii) χ^2 approximation.
3. Find confidence intervals for $0.25, 0.5$ and 0.75-quantiles in the previous example when $n = 50, 200$. Compare with the asymptotic normal confidence intervals for quantiles (can be derived following a similar exercise as in Problem 1).