

Problem I

1. Sample observations y_1, \dots, y_{100} and x_1, \dots, x_{100} from $y_i | x_i = f(x_i) + \epsilon_i, \epsilon_i \sim N(0, 0.5), x_i \sim U(0, 1), f \sim GP(0, c), c(x, x') = e^{-(x-x')^2}$.
2. Fit the model (based on the first 50 observations (training data)) $y_i = f(x_i) + \epsilon_i, \epsilon_i \sim N(0, \sigma^2) f \sim GP(0, c), c = e^{-\kappa(x-x')^2}, \sigma^2 \sim IG(a, b)$ and an *appropriate* discrete uniform prior on κ
3. Plot the posterior predictive mean and the 95 % point-wise credible intervals for the next 50 observations (test data) and estimate the coverage probability.
4. Assess sensitivity on the mean squared prediction error and the coverage probability with respect to the hyperparameters for the priors for σ^2 and κ .