**7.33** We wish to test the hypothesis $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$, where $\mu =$ true mean daily iron intake for 9–11-year-old boys below the poverty level and $\mu_0 =$ true mean daily iron intake for 9–11-year-old boys in the general population.

**7.34** We must use a one-sample $t$ test. We reject $H_0$ if $t < t_{n-1,\alpha/2}$, or $t > t_{n-1,1-\alpha/2}$, where $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$, and accept $H_0$ otherwise. We have that $\mu_0 = 14.44$, $n = 51$, $\alpha = .05$, $\overline{x} = 12.50$, $s = 4.75$. Therefore,

$$
\begin{aligned}
t &= \frac{12.50 - 14.44}{4.75/\sqrt{51}} \\
&= \frac{-1.94}{0.665} = -2.917
\end{aligned}
$$

The critical values are $t_{50,.025}$ and $t_{50,.975}$. Since $t < t_{40,.025} = -2.021 < t_{50,.025}$, it follows that we reject $H_0$ at the 5% level. We conclude that 9–11-year-old boys below the poverty level have a significantly lower mean iron intake than comparably aged boys in the general population.

**7.35** To obtain the $p$-value, we must compute $2 \times Pr(t_{50} > 2.917)$. Since $t_{40,.995} = 2.704$, $t_{40,.9995} = 3.551$ and $2.704 < 2.917 < 3.551$, if we had 40 *df*, then $2 \times (1-.9995) < p < 2 \times (1-.995)$ or $.001 < p < .01$. Similarly, since $t_{60,.995} = 2.660$, $t_{60,.9995} = 3.460$ and $2.660 < 2.917 < 3.460$, if we had 60 *df*, it would also follow that $.001 < p < .01$. Therefore, since we actually have 50 *df*, and we reach the same conclusion with either 40 or 60 *df*, it follows that $.001 < p < .01$. The exact $p$-value obtained by computer is $p = 2 \times Pr(t_{50} > 2.917) = .005$.

**7.36** The hypotheses to be tested are $H_0: \sigma^2 = \sigma_0^2$ versus $H_1: \sigma^2 \neq \sigma_0^2$ where $\sigma^2$ = underlying variance in low-income population, $\sigma_0^2 =$ underlying variance in the general population.

**7.37** We use a one-sample chi-square test. We reject $H_0$ if $X^2 = \dfrac{(n-1)s^2}{\sigma_0^2} < \chi^2_{n-1,\alpha/2}$ or $X^2 > \chi^2_{n-1,1-\alpha/2}$.

We have $\sigma_0^2 = 5.56^2 = 30.91$, $n = 51$, $\alpha = .05$, $s^2 = 4.75^2 = 22.56$. Thus,

$$
\begin{aligned}
X^2 &= \frac{(n-1)s^2}{\sigma_0^2} \\
&= \frac{50(4.75)^2}{5.56^2} = 36.49 \sim \chi^2_{50} \text{ under } H_0
\end{aligned}
$$

The critical values are $\chi^2_{50,.025} = 32.36$ and $\chi^2_{50,.975} = 71.42$. Since $32.36 < 36.49 < 71.42$, it follows that we accept $H_0$ at the 5% level and conclude that there is no significant difference between the variance of iron intake for the low-income population and the general population.

**7.38** Since $\chi^2_{50,.05} = 34.76$, $\chi^2_{50,.10} = 37.69$ and $34.76 < 36.49 < 37.69$, it follows that $2 \times .05 < p < 2 \times .10$ or $.10 < p < .20$. The exact $p$-value obtained by computer is $p = 2 \times Pr\left(\chi^2_{50} < 36.49\right) = .15$.

**7.39** A 95% confidence interval for the underlying variance $\left(\sigma^2\right)$ is given by

$$\begin{aligned}\left[\frac{(n-1)s^2}{\chi^2_{n-1,.975}}, \frac{(n-1)s^2}{\chi^2_{n-1,.025}}\right] &= \left[\frac{50(4.75)^2}{\chi^2_{50,.975}}, \frac{50(4.75)^2}{\chi^2_{50,.025}}\right] \\ &= \left(\frac{1128.125}{71.42}, \frac{1128.125}{32.36}\right) \\ &= (15.80, 34.86)\end{aligned}$$

Since this confidence interval contains $\sigma_0^2 = 5.56^2 = 30.91$, we conclude that the underlying variance of the low-income population is not significantly different from that of the general population.

**7.40** The inferences made with the hypothesis-testing approach in Problems 7.37 and 7.38 are the same as those made with the CI approach in Problem 7.39, viz. there is no significant difference between the variance of iron intake for the low income population and the variance of the general population.

**7.85** Let $X$ = number of subjects with side effects. Under $H_0$, $X$ will be binomially distributed with parameters $n = 10$ and $p = .2$. The type I error $= \alpha = \Pr\left(X \ge 4\right)$. Thus,

$$\alpha = \Pr\left[X \ge 4 \middle| X \sim \text{binomial}(10, .2)\right].$$

From Table 1 (Appendix, text), $\alpha = .0881 + .0264 + .0055 + .0008 + .0001 = .1209$. Thus, the type I error = 12%.

**7.86** The power of the test $= 1 - \beta = \Pr\left[X \ge 4 \middle| \text{binomial}(10, .5)\right]$. From Table 1,
$1 - \beta = .2051 + .2461 + .2051 + .1172 + .0439 + .0098 + .0010 = .8282$.

Thus, the power is 83%.

**7.87** We refer to the sample size formula for the one-sample binomial test given in equation 7.46 (in Chapter 7, text), but we modify the formula since we are using a one-sample test We have

$$N = \frac{p_0 q_0 \left( z_{1-\alpha} + z_{1-\beta} \sqrt{\frac{p_1 q_1}{p_0 q_0}} \right)^2}{\left(p_1 - p_0\right)^2}$$

In this case, $p_0 = .2,\ q_0 = .8,\ p_1 = .5 = q_{1},\ z_{1-\alpha} = z_{.8791} = 1.17$, $z_{1-\beta} = z_{.99} = 2.326$.
Thus,

$$N = \frac{.2(.8)\left( 1.17 + 2.326\sqrt{\frac{.5(.5)}{.2(.8)}} \right)^2}{(.2-.5)^2}$$

$$= \frac{.16(4.0775)^2}{0.09} = 29.6$$

Thus, we need to enroll 30 subjects in the pilot study in order to have 99% power.

**7.87** We refer to the sample size formula for the one-sample binomial test given in equation 7.46 (in Chapter 7, text), but we modify the formula since we are using a one-sample test We have

$$N = \frac{p_0 q_0 \left( z_{1-\alpha} + z_{1-\beta} \sqrt{\frac{p_1 q_1}{p_0 q_0}} \right)^2}{(p_1 - p_0)^2}$$

In this case, $p_0 = .2,\ q_0 = .8,\ p_1 = .5 = q_{1,}\ z_{1-\alpha} = z_{.8791} = 1.17$ , $z_{1-\beta} = z_{.99} = 2.326$ .
Thus,

$$N = \frac{.2(.8)\left( 1.17 + 2.326 \sqrt{\frac{.5(.5)}{.2(.8)}} \right)^2}{(.2 - .5)^2}$$

$$= \frac{.16(4.0775)^2}{0.09} = 29.6$$

Thus, we need to enroll 30 subjects in the pilot study in order to have 99% power.

**8.25** Test the hypothesis $H_0: \mu_d = 0$ versus $H_1: \mu_d \neq 0$, where $\mu_d$ represents the mean difference in 1-hour concentration (drug A – drug B) in a specific person.

**8.26** Use a paired *t* test to test these hypotheses. An independent-samples *t* test cannot be used in this case, because the two samples are from the same people and are not independent.

**8.27** We have the $d_i$ given below

| Difference in 1-hour urine concentration between type A and type B aspirin | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Person ($i$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $d_i$ | 2 | 6 | 3 | 7 | 0 | –2 | 2 | 6 | 5 | 7 |

It follows that $\bar{d} = 3.60$, $s_d = 3.098$. Thus,

$$t = \frac{\bar{d}}{s_d / \sqrt{n}} = \frac{3.60}{3.098 / \sqrt{10}} = \frac{3.60}{0.980} = 3.67 \sim t_9$$

Since $t_{9,.995} = 3.250$, $t_{9,.9995} = 4.781$ based on a two-sided test, it follows that $.001 < p < .01$, and $H_0$ is rejected and we conclude that aspirin A has a significantly higher concentration in urine specimens than aspirin B does. The exact *p*-value, obtained by computer is .005.

**8.28** The best point estimate is $\bar{d} = 3.60$ mg%.

**8.29** A 95% confidence interval is given by

$$\bar{d} \pm t_{9,.975}\left(\frac{s_d}{\sqrt{10}}\right) = 3.60 \pm 2.262\left(\frac{3.098}{\sqrt{10}}\right)$$
$$= (1.38,\ 5.82) \text{ mg\%}$$

**8.30** If the test result had been significant at the 5% level, then the confidence interval would have excluded 0; otherwise, it would have included 0. The former possibility is what actually occurred.

**8.31** Test the hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_1: \sigma_1^2 \neq \sigma_2^2$. Use the *F* test with test statistic

$$F = \frac{s_1^2}{s_2^2} = \left(\frac{7.3}{2.7}\right)^2 = 7.31 \sim F_{39,39} \text{ under } H_0$$

Since $F_{39,39,.975} < F_{24,30,.975} = 2.14 < F$, it follows that $p < .05$, and the variances (and thus the standard deviations) of the two groups are significantly different.

**8.32** Test the hypothesis $H_0: \mu_1 = \mu_2,\ \sigma_1^2 \neq \sigma_2^2$ versus $H_1: \mu_1 \neq \mu_2,\ \sigma_1^2 \neq \sigma_2^2$. Use the two-sample $t$ test with unequal variances because $H_0$ was rejected in Problem 8.31.
We have the test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{11.6 - 6.9}{\sqrt{\frac{7.3^2}{40} + \frac{2.7^2}{40}}} = \frac{4.7}{1.231} = 3.82$$

Compute the appropriate $df\,(d')$ as follows:

$$d' = \frac{\left(\frac{7.3^2}{40} + \frac{2.7^2}{40}\right)^2}{\left(\frac{7.3^2}{40}\right)^2 \Big/ 39 + \left(\frac{2.7^2}{40}\right)^2 \Big/ 39} = \frac{2.294}{0.046} = 49.5$$

Thus, there are $d'' = 49$ $df$. Since $t = 3.82 > t_{40,.975} = 2.021 > t_{49,.975}$, it follows that $p < .05$ and $H_0$ is rejected at the 5% level. Thus, there is a significant difference between the mean CO concentrations in the two working environments.

**8.33** A 95% CI for the true mean difference in CO is given by

$$\bar{x}_1 - \bar{x}_2 \pm t_{d'',.975}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 4.7 \pm t_{49,.975}(1.231)$$

Using MINITAB, we approximate $t_{49,.975}$ by 2.009.
Therefore, the 95% CI is $4.7 \pm 2.009(1.231) = 4.7 \pm 2.47 = (2.2,\ 7.2)$.