

Problem I

Write down the steps for a Gibbs sampler for signal estimation using Horseshoe prior in following linear model. Let $Y_{n \times 1}$ and $X_{n \times p}$ be the response vector and the covariate matrix respectively.

$$\begin{aligned} Y | \beta &\sim N(X\beta, I_n) \\ \beta_j | \lambda_j, \tau &\sim N(0, \lambda_j^2 \tau^2), \\ \lambda_j &\sim \text{Ca}^+ \\ \tau &\sim \text{Ca}^+ \end{aligned}$$

where Ca^+ is a half-Cauchy distribution on the positive real line with density $2/(\pi(1+x^2)), x > 0$.