Homework 8 Solutions

Problems

5.21

$$P\{X > 74\} = P\left(\frac{X-71}{\sqrt{6.25}} > \frac{74-71}{\sqrt{6.25}}\right)$$
$$= P(Z > 1.2)$$
$$= \phi(-1.2)$$
$$= 1 - 0.8849$$
$$= 0.1151$$

$$P\{X > 77 \mid X > 72\} = \frac{P(X>77, X>72)}{P(X>72)}$$

$$= \frac{P(X>77)}{P(X>72)}$$

$$= \frac{P(Z>2.4)}{P(Z>0.4)}$$

$$= \frac{1-0.9918}{1-0.6554}$$

$$= \frac{0.0082}{0.3446}$$

$$= 0.2380$$

5.22

A. P{.9000−.005 < X < .9000+.005}

$$= P\left\{-\frac{.005}{.003} < Z < \frac{.005}{.003}\right\}$$

$$= P\{-1.67 < Z < 1.67\}$$

$$= 2 * \Phi(1.67) - 1$$

$$= .9050.$$

Hence 9.5 percent will be defective (that is each will be defective with probability 1 − .9050 = .0950).

B. $P\left\{-\frac{0.005}{\sigma} < Z < \frac{0.005}{\sigma}\right\} = 2 * \phi\left(\frac{0.005}{\sigma}\right) - 1$
$= 0.99$

when $\phi\left(\frac{0.005}{\sigma}\right) = 0.995 \rightarrow \frac{0.005}{\sigma} = 2.575 \rightarrow \sigma = 0.0019$

5.24

With C denoting the life of a chip, and Φ the standard normal distribution function we have

$$P\{C < 1.8\text{x}10^6\} = \Phi\left(\frac{1.8\text{x}10^6 - 1.4\text{x}10^6}{3\text{x}10^5}\right)$$
$$= \Phi(1.33)$$
$$= 0.9082$$

Thus, if N is the number of the chips whose life is less than 1.8×10^6, then N is a binomial random variable with parameters (100, .9082).
Hence,

$$P\{N > 19.5\} \approx 1 - \Phi\left(\frac{19.5 - 90.82}{90.82(.0918)}\right) = 1 - \Phi(-24.7) \approx 1.$$

5.25

Let X denote the number of unacceptable items among the next 150 produced. Since X is a binomial random variable with mean 150(.05) = 7.5 and variance 150(.05)(.95) = 7.125, we obtain that, for a standard normal random variable Z.

$$P\{X \leq 10\} = P\{X \leq 10.5\}$$

$$= P\left(\frac{X-7.5}{\sqrt{7.125}} \leq \frac{10.5-7.5}{\sqrt{7.125}}\right)$$

$$= P(Z \leq 1.1239)$$
$$= 0.8695$$

The exact result can be obtained by using the text diskette, and (to four decimal places) is equal to .8678.

5.32

A. $P\{X > 2\} = e^{-1}$

B. $P\{X \geq 10 | X > 9) = e^{-1/2}$

5.33

$P\{X > 8\} = e^{-1}$

5.37

A. $P\left\{|X| > \frac{1}{2}\right\} = P\left\{X > \frac{1}{2}\right\} + P\left\{X < -\frac{1}{2}\right\} = \frac{1}{2}$

B. $P\{|X| \leq a\} = P\{-a \leq X \leq a\} = a$, 0<a<1

Therefore,

$f_{|X|}(a) = 1$, 0<a<1 .

That is, |X| is uniform on (0,1).

5.39

$$F_Y(y) = P\{\log(X) \le y\}$$
$$= P\{X \le e^y\}$$
$$= F_X(e^y)$$

Then,

$$f_Y(y) = f_X(e^y) * e^y$$
$$= e^y e^{-e^y}$$

5.40

$$F_Y(y) = P\{e^X \le y\} = F_X(\log y)$$

Then,

$f_Y(y) = f_X(logy) * \frac{1}{y} = \frac{1}{y}$, 1<y<e

Theoretical Exercises

5.9

The final step of parts (a) and (b) use that –Z is also a standard normal random variable.

A. $P\{Z > x\} = P\{-Z < -x\} = P\{Z < -x\}$

B. $$\begin{aligned} P\{|Z| > x\} &= P\{Z > x\} + P\{Z < -x\} \\ &= P\{Z > x\} + P\{-Z > x\} \\ &= 2 * P\{Z > x\} \end{aligned}$$

C. $P\{|Z| < x\} = 1 - P\{|Z| > x\}$

$= 1 - 2 * P\{Z > x\}$ (by B)

$= 1 - 2(1 - P\{Z < x\})$

$= 1 - 2 + 2\, P\{Z < x\}$

$= 2\, P\{Z < x\} - 1$