Jointly distributed random variables - Class 22

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1 Sum of two independent random variables

1. $X \sim \text{Bin}(n, p)$, $Y \sim Bin(m, p)$, then $X + Y \sim Bin(m + n, p)$
2. $X \sim Poiss(\lambda), Y \sim Poiss(\mu), X + Y \sim Poiss(\lambda + \mu)$
3. If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, calculate the conditional distribution of X, given that $X + Y = n$ and Y, given that $X + Y = n$. These will be $Bin(n, \frac{\lambda_1}{\lambda_1+\lambda_2})$ and $Bin(n, \frac{\lambda_2}{\lambda_1+\lambda_2})$ respectively.
4. Suppose that the number of people that enter a post office on a given day is a Poisson random variable with parameter λ. Show that if each person that enters the post office is a male with probability p and a female with probability 1-p, then the number of males and females entering the post office are independent Poisson random variables with respective parameters λp and $\lambda(1 - p)$.

   (Let X and Y denotes the number of males and females that enter the post office. Then

$$\begin{aligned} P(X = i, Y = j) &= P(X = i, Y = j \mid X + Y = i + j)P(X + Y = i + j) + \\ &\quad P(X = i, Y = j \mid X + Y \neq i + j)P(X + Y \neq i + j) \\ &= P(X = i, Y = j \mid X + Y = i + j)P(X + Y = i + j) \end{aligned}$$

   Since $(X + Y) \sim Poiss(\lambda), P(X + Y = i + j) = \exp\{-\lambda\}\lambda^{i+j}/(i + j)!$ and

$$P(X = i, Y = j \mid X + Y = i + j) = \binom{i+j}{i} p^i(1 - p)^j$$

   Hence

$$\begin{aligned} P(X = i, Y = j) = \binom{i+j}{i} p^i(1 - p)^j \exp\{-\lambda\}\lambda^{i+j}/(i + j)! \\ = e^{-\lambda p}\frac{(\lambda p)^i}{i!} e^{-\lambda(1-p)}\frac{(\lambda(1 - p))^j}{j!} \end{aligned}$$

   Hence $X \sim Poisson(\lambda p)$ and $Y \sim Poisson(\lambda(1 - p))$.)