STA 4442/5440 Midterm 2

October 31, 2012

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

Pledge: I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

Signature: ____

INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 70 minutes (11:05 A.M to 12:15 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 5 problems and is worth 80 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.
1	20	
2	10	
3	20	
4	15	
5	15	

TOTAL: __________

Question 1. Independent trials, each resulting in a success with probability $\frac{2}{3}$, are performed 4 times. Let X be the total number of successes and $Y = \sin\left(\frac{\pi}{2}X\right)$.

(a) (12 points) Find $P(X \geq 1)$, expectation and variance of X.

(b) (8 points) Find the expectation of Y.
(Use $\sin\left(0\right) = \sin\left(\pi\right) = \sin\left(2\pi\right) = 0, \sin\left(\pi/2\right) = 1, \sin\left(3\pi/2\right) = -1$)

Question 2. (10pts.) While checking the galley proofs of a book, the authors found 1.6 printer's errors per page on average. Assuming printing errors to be independent across pages, what is the probability that in 4 consecutive pages, there are no errors on the first and third pages, and one error on each of the other two? (Hint: Use Poisson distribution)

Question 3. (20 pts.) An examination is often regarded as being good if the test scores of those taking the examination can be approximated by a normal density function. The instructor often uses the test scores to estimate the normal parameters μ and σ^2 and then assign the letter grade A to those whose test score is greater than $\mu+\sigma$, B to those whose score is between μ and $\mu+\sigma$, C to those whose score is between $\mu-\sigma$ and μ, D to those whose score is between $\mu-2\sigma$ and $\mu-\sigma$, and F to those getting a score below $\mu-2\sigma$. This is sometimes referred to as grading "on the curve". Find the probabilities that

(a) a student gets a grade A

(b) a student gets a grade B

(c) a student gets a grade C

(d) a student gets a grade D

(e) a student gets a grade F

(Given $1-\Phi(1)=0.1587$, $\Phi(1)-\Phi(0)=0.3413$, $\Phi(2)-\Phi(1)=0.1359$, $\Phi(-2)=0.0228$, where $\Phi(\cdot)$ denotes the standard normal cdf).

Question 4. (15 pts.) Suppose the pdf of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, 0 \leq x \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

a) Find the cdf $F(x)$.

b) $P(1 \leq X \leq 1.5)$.

Question 5. (15 pts.) You arrive at a bus stop at 10 am, knowing that the bus will arrive at some time uniformly distributed between 10 am and 10:30 am.

(a) Find the probability that you will have to wait longer than 10 minutes.

(b) If at 10:15 am the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?