

## STA 4442/5440 Midterm 2 Practice 1

November 7, 2012

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

**Pledge:** I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

**Signature:** \_\_\_\_\_

### INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 70 minutes (11:05 A.M to 12:15 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 6 problems and is worth 80 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.
1	20	
2	10	
3	20	
4	10	
5	10	
6	10	

**TOTAL:** \_\_\_\_\_

**Question 1.** (20 pts.) In actuarial science, one of the models used for describing mortality is

$$f(x) = \begin{cases} Cx^2(100 - x)^2, & 0 \leq x \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

where  $x$  denotes the age at which a person dies.

- (a) Find the value of  $C$ .
- b) Let  $A$  be the event “Person lives past 60.” Find  $P(A)$ .
- c) Find the expected mortality.

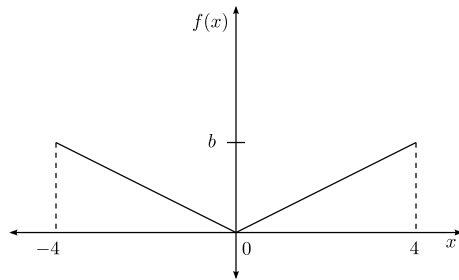
**Question 2.** (10 pts.)  $X$  and  $Y$  are two discrete random variables taking values  $-1, 0$  and  $+1$  each with joint probability given by

Table 1: Joint probability Table

$Y \downarrow X \rightarrow$	-1	0	+1	Total
-1	0	1/4	0	1/4
0	1/4	0	1/4	1/2
+1	0	1/4	0	1/4
Total	1/4	1/2	1/4	1

- a) Find marginal p.m.f of  $X$  and  $Y$ .
- b) Find whether  $X$  and  $Y$  are independent or not.

**Question 3.** (20 pts.) The figure is the probability density curve of the random variable  $X$ .



- a) Find  $b$  so that  $f(x)$  is a probability density function.
- b) What is  $P(-4 \leq X \leq 3)$ ?
- c) What is  $P(X = 1)$ ?
- d) What is  $E(X)$ ?

**Question 4.** (10 pts.) Two species are competing in a region for control of a limited amount of a certain resource. Let  $X$  =proportion of resource controlled by one species and suppose  $X \sim \text{Unif}([0, 1])$ . Let  $h(X) = \max(X, 1 - X)$ , then  $h(X)$  is the amount of resource controlled by the superior species.

a) Find  $E(h(X))$ .

b) Find  $\text{Var}(h(X))$ .

**Question 5.** (10 pts.) Buses arrive at a specified stop at 15-minute intervals starting at 7 a.m. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that she waits

(a) less than 5 minutes for a bus.

(b) more than ten minutes for a bus.

**Question 6.** (10 pts.) A point is picked randomly from the interval  $[0, L]$ .

(a) Define suitably a random variable  $X$  denoting the ratio of the length of the shorter and the longer interval formed.

(b) Find  $P(X > 0.5)$ .

(c) Find  $E(X)$ .