

## STA 4442/5440 Midterm 2 Practice 2

November 7, 2012

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

**Pledge:** I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

**Signature:** \_\_\_\_\_

### INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 70 minutes (11:05 A.M to 12:15 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 5 problems and is worth 80 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.
1	20	
2	10	
3	20	
4	15	
5	15	

**TOTAL:** \_\_\_\_\_



**Question 1.** Independent trials, each resulting in a success with probability  $\frac{2}{3}$ , are performed 4 times. Let  $X$  be the total number of successes and  $Y = \sin\left(\frac{\pi}{2}X\right)$ .

(a) (12 points) Find  $P(X \geq 1)$ , expectation and variance of  $X$ .

(b) (8 points) Find the expectation of  $Y$ .

(Use  $\sin(0) = \sin(\pi) = \sin(2\pi) = 0$ ,  $\sin(\pi/2) = 1$ ,  $\sin(3\pi/2) = -1$ )

**Question 2.** (10pts.) While checking the galley proofs of a book, the authors found 1.6 printer's errors per page on average. Assuming printing errors to be independent across pages, what is the probability that in 4 consecutive pages, there are no errors on the first and third pages, and one error on each of the other two? (Hint: Use Poisson distribution)

**Question 3.** (20 pts.) An examination is often regarded as being good if the test scores of those taking the examination can be approximated by a normal density function. The instructor often uses the test scores to estimate the normal parameters  $\mu$  and  $\sigma^2$  and then assign the letter grade  $A$  to those whose test score is greater than  $\mu + \sigma$ ,  $B$  to those whose score is between  $\mu$  and  $\mu + \sigma$ ,  $C$  to those whose score is between  $\mu - \sigma$  and  $\mu$ ,  $D$  to those whose score is between  $\mu - 2\sigma$  and  $\mu - \sigma$ , and  $F$  to those getting a score below  $\mu - 2\sigma$ . This is sometimes referred to as grading “on the curve”. Find the probabilities that

(a) a student gets a grade  $A$

(b) a student gets a grade  $B$

(c) a student gets a grade  $C$

(d) a student gets a grade  $D$

(e) a student gets a grade  $F$

(Given  $1 - \Phi(1) = 0.1587$ ,  $\Phi(1) - \Phi(0) = 0.3413$ ,  $\Phi(2) - \Phi(1) = 0.1359$ ,  $\Phi(-2) = 0.0228$ , where  $\Phi(\cdot)$  denotes the standard normal cdf).

**Question 4.** (15 pts.) Suppose the pdf of a continuous random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Find the cdf  $F(x)$ .

b)  $P(1 \leq X \leq 1.5)$ .

**Question 5.** (15 pts.) You arrive at a bus stop at 10 am, knowing that the bus will arrive at some time uniformly distributed between 10 am and 10:30 am.

(a) Find the probability that you will have to wait longer than 10 minutes.

(b) If at 10:15 am the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?