STA 4442/5440 Midterm 2 Practice 2

November 7, 2012

Name:

FSUID:

Please sign the following pledge and read all instructions carefully before starting the exam.

Pledge: I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code.

Signature: ____

INSTRUCTIONS:

- This is a closed-book, closed-notes exam. You may **not** refer to your notes, the text, or any other books. You may use a calculator.
- Total time is 70 minutes (11:05 A.M to 12:15 P.M.)
- **Show all work**, clearly and in order, if you want to receive full credit. When you use your calculator, explain all relevant mathematics. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- **Answer all the questions in the space provided. You may attach additional sheets if necessary.**
- This test has 5 problems and is worth 80 points. It is your responsibility to make sure that you have all of the problems.
- **Good luck!**

Prob. No.	Max Points	Earned Pts.
1	20	
2	10	
3	20	
4	15	
5	15	

TOTAL: __________

Question 1. Independent trials, each resulting in a success with probability $\frac{2}{3}$, are performed 4 times. Let X be the total number of successes and $Y = \sin\left(\frac{\pi}{2}X\right)$.

(a) (12 points) Find $P(X \geq 1)$, expectation and variance of X.

(b) (8 points) Find the expectation of Y.
(Use $\sin(0) = \sin(\pi) = \sin(2\pi) = 0, \sin(\pi/2) = 1, \sin(3\pi/2) = -1$)

a) $X \sim \text{Bin}(4, 2/3)$. $P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{4}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^4$

$= 1 - \left(\frac{1}{3}\right)^4$

$E(X) = 4 \cdot 2/3 = 8/3$

$V(X) = 4 \cdot 2/3 \cdot 1/3 = \frac{8}{9}$

b) $E(Y) = E\left(\sin\left(\frac{\pi}{2}X\right)\right)$

$= \sin\left(\frac{\pi}{2}\cdot 0\right)P(X=0) + \sin\left(\frac{\pi}{2}\cdot 1\right)P(X=1)$

$+ \sin\left(\frac{\pi}{2}\cdot 2\right)P(X=2) + \sin\left(\frac{\pi}{2}\cdot 3\right)P(X=3)$

$+ \sin\left(\frac{\pi}{2}\cdot 4\right)\cdot P(X=4)$

$= P(X=1) - P(X=3)$

$= \binom{4}{1}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 - \binom{4}{3}\left(\frac{2}{3}\right)^3\frac{1}{3}$

$= \frac{4}{1}\cdot\frac{2}{3}\cdot\frac{1}{3}\left[\left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right]$

$= 4\cdot\frac{2}{3}\cdot\frac{1}{3}\left(-\frac{3}{9}\right) = -\frac{8}{27}$

Question 2. (10pts.) While checking the galley proofs of a book, the authors found 1.6 printer's errors per page on average. Assuming printing errors to be independent across pages, what is the probability that in 4 consecutive pages, there are no errors on the first and third pages, and one error on each of the other two? (Hint: Use Poisson distribution)

X_i = # of misprints in page i

$X_i \sim \text{Poisson}(1.6)$ $\qquad P(X_i = k) = \dfrac{e^{-1.6} (1.6)^k}{k!}$

$P(X_1 = 0, X_2 = 1, X_3 = 0, X_4 = 1)$

$= P(X_1 = 0)\, P(X_2 = 1)\, P(X_3 = 0)\, P(X_4 = 1)$ (By independence

$= 0.2019 \times 0.3230 \times 0.2019 \times 0.3230$

$= 0.0043$

Question 3. (20 pts.) An examination is often regarded as being good if the test scores of those taking the examination can be approximated by a normal density function. The instructor often uses the test scores to estimate the normal parameters μ and σ^2 and then assign the letter grade A to those whose test score is greater than $\mu+\sigma$, B to those whose score is between μ and $\mu+\sigma$, C to those whose score is between $\mu-\sigma$ and μ, D to those whose score is between $\mu-2\sigma$ and $\mu-\sigma$, and F to those getting a score below $\mu-2\sigma$. This is sometimes referred to as grading "on the curve". Find the probabilities that

(a) a student gets a grade A

(b) a student gets a grade B

(c) a student gets a grade C

(d) a student gets a grade D

(e) a student gets a grade F

(Given $1-\Phi(1)=0.1587$, $\Phi(1)-\Phi(0)=0.3413$, $\Phi(2)-\Phi(1)=0.1359$, $\Phi(-2)=0.0228$, where $\Phi(\cdot)$ denotes the standard normal cdf).

X = Score , $X \sim N(\mu, \sigma^2)$

a) $P(X > \mu+\sigma) = P\left(\frac{X-\mu}{\sigma} > 1\right) = 1-\Phi(1) = 0.1587$

b) $P(\mu < X < \mu+\sigma) = P\left(0 < \frac{X-\mu}{\sigma} < 1\right)$

$= \Phi(1) - \Phi(0) = 0.3413$

c) $P(\mu-\sigma < X < \mu) = \Phi(0) - \Phi(-1)$

$= \Phi(0) - (1-\Phi(1))$

$= \Phi(1) - \Phi(0) = 0.341$

d) $P(\mu-2\sigma < X < \mu-\sigma)$

$= \Phi(2) - \Phi(1) = 0.1359$

e) $\Phi(-2) = 0.0228$

Question 4. (15 pts.) Suppose the pdf of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, 0 \le x \le 2 \\ 0, \text{otherwise} \end{cases}$$

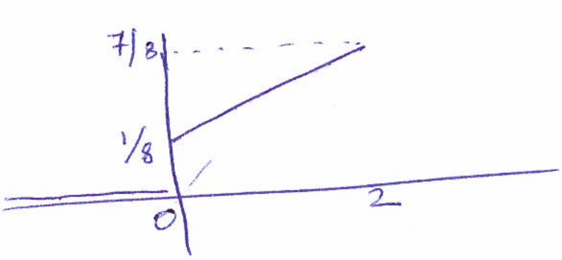
$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

a) Find the cdf $F(x)$.

b) $P(1 \le X \le 1.5)$.

a) $F(x) = \begin{cases} 0, \text{ if } x < 0 \\ \frac{x}{8} + \frac{3}{16}x^2, 0 \le x \le \\ 1, x > 2 \end{cases}$

$f(x) =$



b) $P(1 \le X \le 1.5)$

$$= \int_{1}^{3/2} \left(\frac{1}{8} + \frac{3}{8}x\right) dx$$

$$= \frac{1}{8}\left(\frac{3}{2} - 1\right) + \frac{3}{8 \times 2}\left(\frac{9}{4} - 1\right)$$

$$= \frac{1}{16} + \frac{3}{16} \cdot \frac{5}{4} = \frac{19}{64}$$

Question 5. (15 pts.) You arrive at a bus stop at 10 am, knowing that the bus will arrive at some time uniformly distributed between 10 am and 10:30 am.

(a) Find the probability that you will have to wait longer than 10 minutes.

(b) If at 10:15 am the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

X : arrival time of Bus $\sim U(10\text{ am}, 10{:}30\text{ a.m})$

Scaling down $X \sim U([0, 30])$

a) $P(X > 10) = \frac{20}{30} = \frac{2}{3}$

b) $P(X > 10{:}25 \mid X > 10{:}15)$

$$= \frac{P((X > 10{:}25) \cap (X > 10{:}15))}{P(X > 10{:}15)}$$

$$= \frac{P(X > 10{:}25)}{P(X > 10{:}15)} = \frac{\frac{5}{30}}{\frac{15}{30}} = \frac{1}{3}$$