Hypothesis testing

One sample χ^2 test for the variance or a normal distribution

1. An arteriosonde machine "prints" blood-pressure readings on a tape so that the measurement can be read rather than heard.

2. An argument for using such a machine is that the variability of measurement obtained by different observers on the same person will be lower than with a standard blood-pressure cuff.

3. Suppose we know from previously published work that $\sigma^2 = 35$ for d_i obtained from the standard cuff. We want to test the hypothesis

$$H_0 : \sigma^2 = \sigma_0^2 = 35 \, vs. \, \sigma^2 \neq \sigma_0^2$$

   How should we perform this test?

4.

$$\begin{aligned} X^2 &= \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{n-1} \\ P(X^2 < \chi^2_{n-1,\alpha/2}) &= \alpha/2 = P(X^2 > \chi^2_{n-1,1-\alpha/2}) \end{aligned}$$

5. If $X^2 < \chi^2_{n-1,\alpha/2}$ or $X^2 > \chi^2_{n-1,1-\alpha/2}$, then H_0 is rejected.

6. $\chi^2_{n-1,\alpha/2} < X^2 < \chi^2_{n-1,1-\alpha/2}$, then H_0 is accepted.

7. Recall the statistic is $X^2 = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{n-1}$.

8. If $S^2 \leq \sigma_0^2$, then p-value $= 2 \times$ (area to the left of X^2 under a χ^2_{n-1} distribution).

9. If $S^2 > \sigma_0^2$, then p-value $= 2 \times$ (area to the of X^2 under a χ^2_{n-1} distribution).

Acceptance and rejection regions for the one-sample χ^2 test for the variance of a normal distribution (two-sided alternative)

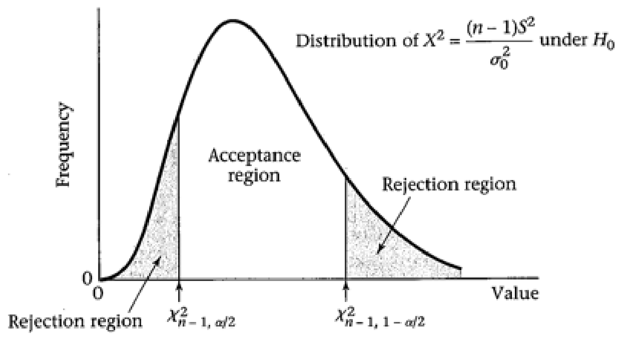
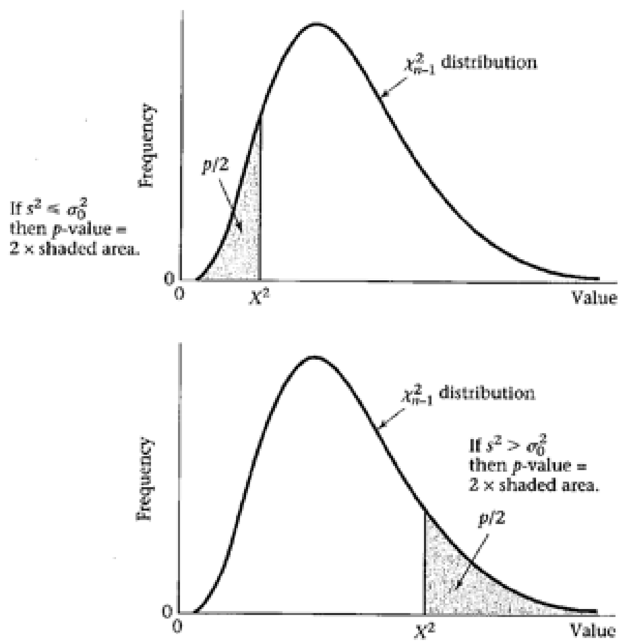
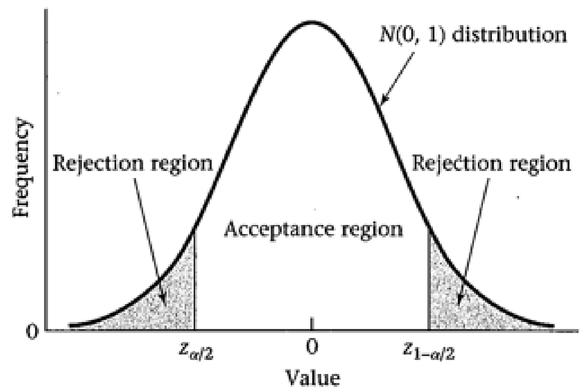


Illustration of the p-value for a one-sample χ^2 test for the variance of a normal distribution (two-sided alternative)



Acceptance and rejection regions for the one-sample binomial test—normal-theory method (two-sided alternative)



One sample test for binomial proportion

Recall that the hypothesis we are testing

$$H_0 : p = p_0,\ vs.\ H_1 : p \neq p_0$$

Let the test statistic be $Z = (\hat{p} - p_0)/\sqrt{p_0 q_0 / n}$. If $Z_{\alpha/2} \leq Z \leq Z_{1-\alpha/2}$, then H_0 is accepted. This test should only be used if $np_0 q_0 \geq 5$.

If $\hat{p} < p_0$, the p-value $= 2 \times \Phi(Z) =$ twice the area to the left of Z under a $N(0,1)$ curve. If $\hat{p} \geq p_0$, then p-value $= 2(1 - \Phi(Z)) =$ twice the area to the right of Z under a $N(0,1)$ curve.

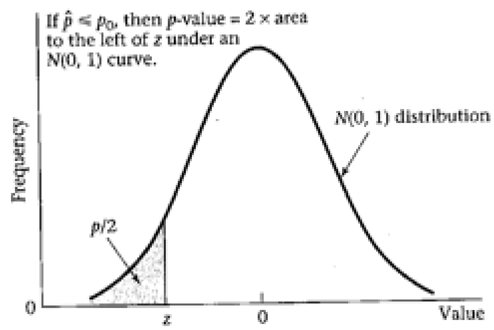
Power and sample size calculation for one sample test for binomial proportion

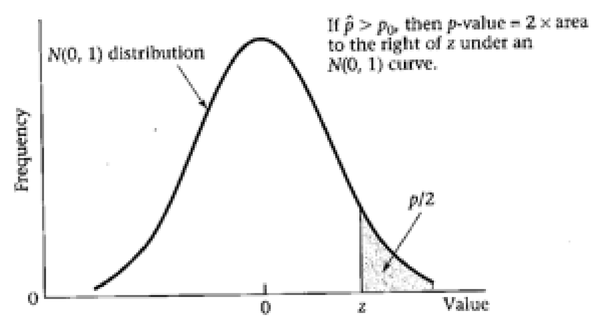
Power for the specific alternative $p = p_1$ is given by

$$\Phi\left[\sqrt{\frac{p_0 q_0}{p_1 q_1}}\left(Z_{\alpha/2} + \frac{|p_0 - p_1|\sqrt{n}}{\sqrt{p_0 q_0}}\right)\right]$$

To use this formula, we assume that $np_0 q_0 \geq 5$ so that the normal theory approximation works.

Illustration of the p-value for a one-sample binomial test—normal-theory method (two-sided alternative)





Sample size estimation for the one-sample binomial test with wo sided alternative

Suppose we wish to test $H_0 : p = p_0$ vs. $H_1 : p \neq p_0$. The sample size needed to conduct a two-sided test with significance level α and power $1 - \beta$ versus the specific alternative hypothesis $p = p_1$ is

$$n = \frac{p_0 q_0 \left(Z_{1-\alpha/2} + Z_{1-\beta}\sqrt{\frac{p_1 q_1}{p_0 q_0}}\right)^2}{(p_1 - p_0)^2}$$

One-Sample Inference for the Poisson Distribution

Many studies have looked at possible health hazards faced by rubber workers. In one such study, a group of 8418 white male workers ages 40-84 (either active or retired) on January 1, 1964, were followed for 10 years for various mortality outcomes. Their mortality rates were then compared with US white male mortality rates in 1968. In one of the reported findings, 4 deaths due to Hodgkins disease were observed compared with 3.3 deaths expected from US mortality rates. Is this difference significant?

Let X be a Poisson random variable with expected value μ. To test the hypothesis $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$ using a two sided test with significance level α,

1. obtain the two-sided $100(1 - \alpha)\%$ confidence interval for μ based on the observed value x of X. Denote the confidence interval by $[c_1, c_2]$.

2. if $\mu_0 < c_1$ or $\mu_0 > c_2$, then reject H_0. If $c_1 \leq \mu_0 \leq c_2$, the accept H_0.

Summary

Figure 7.18 Flowchart for appropriate methods of statistical inference