Continuous Probability Distributions

Continuous Random Variables

- Discrete random variables
  - Random variables whose set of possible values is either finite or countably infinite.
- Continuous random variables
  - Random variables whose set of possible values is uncountable.
  - The blood pressure of a patient.
  - The amount of precipitation in a year.

Probability Density Function

- Continuous probability distribution
  - If there exists a nonnegative function f, defined for all real $x \in (-\infty, \infty)$, having the property that for any set B of real numbers

$$P(X \in B) = \int_B f(x)dx$$

- The function f is called the probability density function of the random variable X.
  - A function such that the area under the density function curve between any two points a and b is equal to the probability that the random variable X falls between a and b.

- The probability of the whole sample space is 1.

$$1 = P(X \in (-\infty, \infty)) = \int_{-\infty}^{\infty} f(x)dx$$

- The probability of a certain region $B = [a, b]$ is

$$P(a \le X \le b) = \int_{a}^{b} f(x)dx$$

- The probability at a certain point is zero

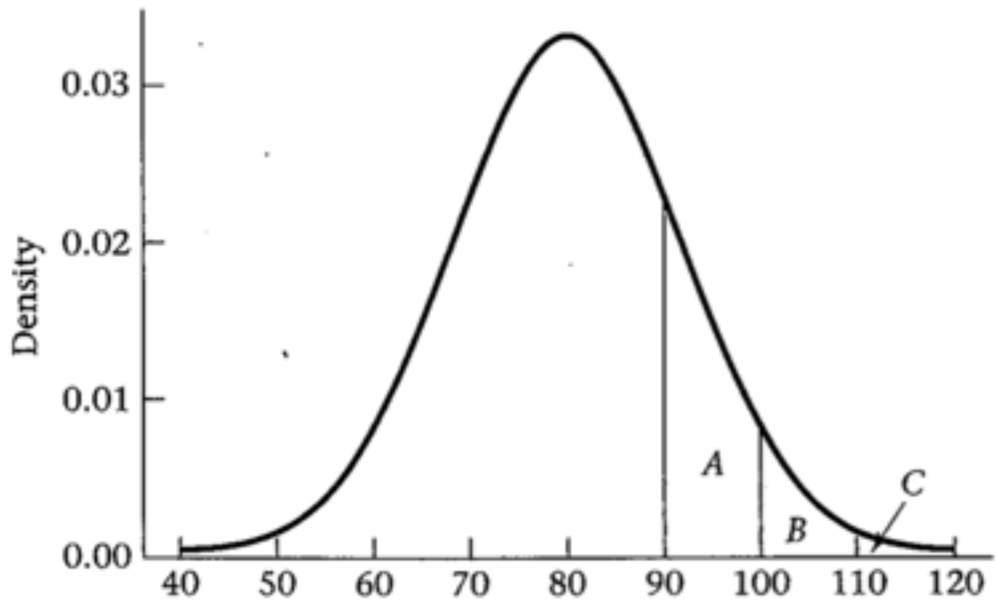
$$P(X = a) = \int_{a}^{a} f(x)dx = 0$$

- The probability around a point a is

$$P(a - \frac{\varepsilon}{2} \le X \le a + \frac{\varepsilon}{2}) = \int_{a-\varepsilon/2}^{a+\varepsilon/2} f(x)dx \approx \varepsilon f(a)$$

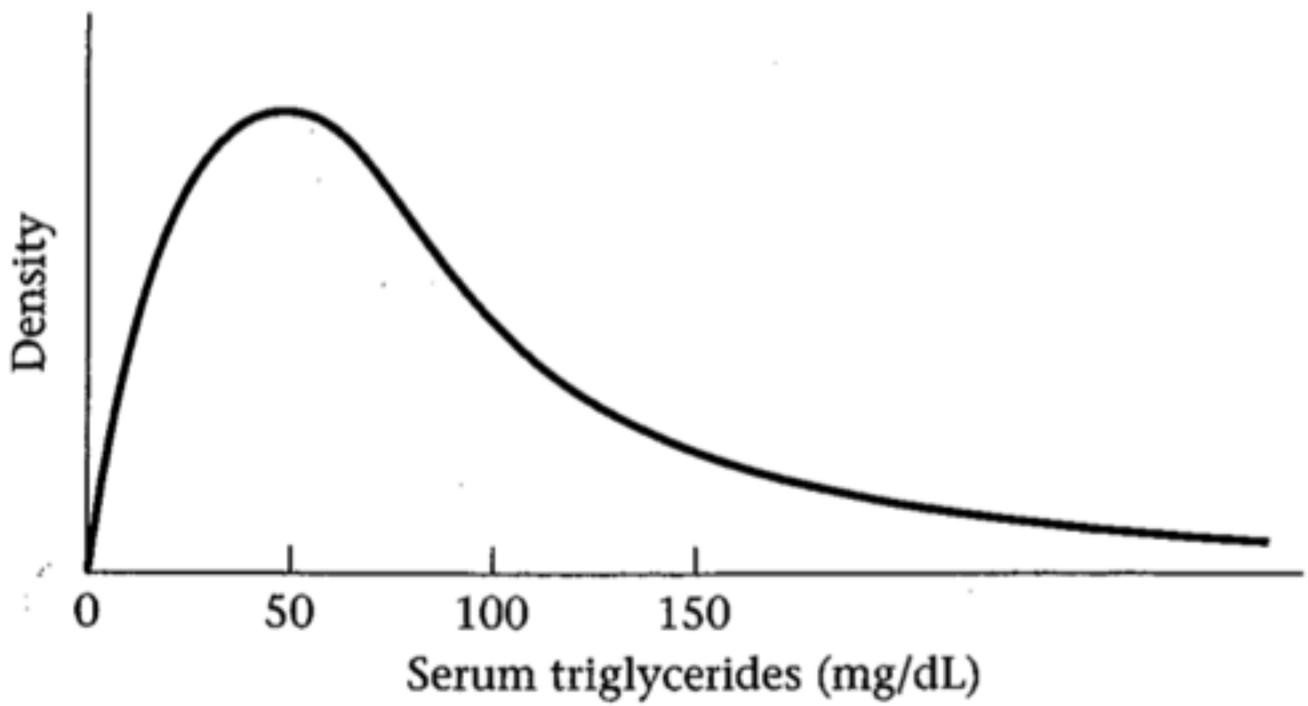
- Ex. Hypertension
- A probability density function for diastolic blood pressure is shown in the following figure. Areas *A*, *B*, and *C* correspond to the probabilities of being mildly hypertensive, moderately hypertensive, and severely hypertensive, respectively.

Figure 5.1 Probability-density function of diastolic blood pressure in 35- to 44-year-old men



- Ex. Cardiovascular Disease
- Serum triglycerides is an asymmetric, positively skewed, continuous random variable.

Figure 5.2 Probability-density function for serum triglycerides



Expected Values of Continuous Random Variables

- Discrete random variable

$$E[X] = \mu = \sum_x xp(x)$$

- Continuous random variable

$$E[X] = \mu = \int_{-\infty}^{\infty} xf(x)dx$$

- If X is a continuous random variable with probability density function $f(x)$, then for any real-valued function g

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

- Find $E[X]$ when the density function of X is

$$f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

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$$E[X] = \int x f(x)\,dx = \int_0^1 2x^2\,dx = 2/3$$

Variance of Continuous Random Variables

- Variance of continuous random variable
  $\mathrm{Var}(X) = \sigma^2 = E[(X - \mu)^2]$

- $\mathrm{Var}(X) = E[X^2] - (E[X])^2$

- $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

- Find Var(X) if X has the pdf

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$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)dx$$

$$= \int_0^1 2x^3 dx$$

$$= 1/2$$

Because $E[X] = 2/3$,

$$\text{Var}(x) = 1/2 - (2/3)^2 = 1/18$$