Expected value of linear combination of random variables

1. The expected value of the sum of n random variables is the sum of n respective expected values.

$$E(L) = E(c_1X_1 + \ldots + c_nX_n) = c_1E(X_1) + c_2E(X_2) + \ldots c_nE(X_n)$$

2. Ex. (Renal Disease) Suppose the expected values of serum creatinine for the white and the black individuals are 1.3 and 1.5 respectively. What is the expected value of the average serum-creatinine level of a single white and a single black individual? The expected value of the average serum-creatinine level $= E(0.5X_1 + 0.5X_2) = 0.5E(X_1) + 0.5E(X_2) = 0.65 + 0.75 = 1.4$

3. Ex. (Renal Disease) Suppose X_1, X_2 are defined as before. If we know that $Var(X_1) = Var(X_2) = 0.25$, then what is the variance of the average serum-creatinine level over a single white and a single black individual? $Var(0.5X_1 + 0.5X_2) = (0.5)^2Var(X_1) + (0.5)^2Var(X_2) = 0.25(0.25) + 0.25(0.25) = 0.125$

4. If X_1, X_2 are defined as in previous examples and are each normally distributed, then what is the distribution of the average $L = 0.5X_1 + 0.5X_2$? Since $E(L) = 1.4, Var(L) = .125$, then $(X_1 + X_2)/2 \sim N(1.4, 0.125)$

Normal Approximation to the Binomial distribution

Ex. Suppose a binomial distribution has parameters $n = 25, p = .4$. How can $P(7 \leq X \leq 12)$ be approximated? $np = 25(.4) = 10, np(1-p) = 25(.4)(.6) = 6.0$. This distribution can be approximated by $N(10, 6)$. $P(7 \leq X \leq 12)$ equals the area under the normal curve from 6.5 to 12.5.

Normal Approximation to the Poisson Distribution

Ex. Consider again the distribution of the number of bacteria in a petri plate of area A. Assume the probability of observing X bacteria is given exactly by a Poisson distribution with parameter $\mu = \lambda A$, where $\lambda = 0.1\, bacteria/cm^2$ and $A = 100\, cm^2$. Suppose 20 bacteria are observed in this area. How unusual is this event? The exact distribution of the number of bacteria observed in $100\, cm^2$ is Poisson with parameter $\mu = 10$. We can approximate this distribution by a normal distribution with mean = 10 and variance = 10