Random Variables - Class 13

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1 Random variables

2 Commonly used random variables

2.1 Bernoulli Random variable

1. Example. Flip a coin. If the outcome is heads, $X = 1$, otherwise $X = 0$.
2. Then X is a Bernoulli r.v., or X follows a Bernoulli distribution.
3. Denition. $P(X = 1) = p, P(X = 0) = 1 - p$, where $p \in (0, 1)$.
4. Then $X \sim Bernoulli(p)$
5. $E(X) = p$, $Var(X) = p(1 - p)$

2.2 Binomial Random Variable

1. Example. Flip five coins independently. X : the number of heads. Find the p.m.f. of X.
2. $P(X = 0)$
3. $P(X = k)$
4. Consider a general problem: Perform n independent trials each of which results in a success w.p. p and in a failure w.p. $1 - p$. Let X represent the number of successes that occur in these n trials. What is the pmf of X?
5. X is a binomial random variable with parameters (n, p).

2.2.1 Properties

1. Let $X \sim Bin(n, p)$ with $0 < p < 1$. Then as k goes from $0 \to n$, $P(X = k)$ rst increases monotonically then decreases monotonically, reaching its largest value when k is the largest integer $\leq (n + 1)p$. $\frac{p(k)}{p(k-1)} = \frac{(n-k+1)p}{k(1-p)}$
2. left skewed binomial distribution ($p \approx 0$), right skewed ($p \approx 1$)
3. Let $X \sim B(n, p)$. $E(X) = np, Var(X) = np(1 - p)$

Problems

1. In a gambling game, a player bets on one of the numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appears i times, $i = 1, 2, 3$, then the player wins i units; on the other hand, if the number bet by the player does not appear on any of the dice, then the player loses 1 unit, Is this game fair to the player? If we assume that the dice are fair and act independently of each other, then the number of times that the number bet appears is a binomial random variable with parameter (3, 1/6). Let X denote the players winnings, we have

$$\begin{aligned} P(X = -1) &= bin(3, 1/6, 0) = 125/216, P(X = 1) = bin(3, 1/6, 1) = 75/216, \\ P(X = 2) &= bin(3, 1/6, 2) = 15/216, P(X = 3) = bin(3, 1/6, 3) = 1/216, \end{aligned}$$

$E(X) = -17/216$.

2. Suppose a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads result, find the conditional probability that the first 3 outcomes are H, T, T. (Let X be the # of H's in the 10 experiments. Then $X \sim Bin(10, p)$. We would like to calculate

$$\begin{aligned} P(HTT \mid 6H) &= \frac{P(HTT \cap 6H)}{P(6H)} \\ &= \frac{P(HTT \cap 5H \text{ in remaining 7 flips})}{P(6H)} \\ &= \frac{p(1-p)^2 \binom{7}{5} p^5 (1-p)^2}{\binom{10}{6} p^6 (1-p)^4} \end{aligned}$$