Sample Space and events - Class 3

September 3, 2013 Debdeep Pati

1 Sample Space

It is the set of all possible outcomes of an experiment.

1. Coin flipping $S = \{H, T\}$
2. Flipping two coins $S = \{(H, H), (H, T), (T, H), (T, T)\}$
3. Order of finish in a race among the 7 horses having post positions $\{1, 2, 3, 4, 5, 6, 7\}$, then S = {all 7! permutations of (1, 2, 3, 4, 5, 6, 7) }
4. Measuring the lifetime of a light bulb $S = \{x : 0 \leq x < \infty\}$.

2 Events

An event E is any subset of the sample space S. This is a set consisting some possible outcomes of the experiment.

1. For flipping coin, E = {H} is the event that a head is seen.
2. Flipping two coins, E = {(H, H), (H, T)} is the event that a head appears on the first coin.
3. Light bulb, E =$\{x : 0 \leq x \leq 100\}$ is the event that the light bulb lasts less than 100 hours.

2.1 Form new events from two or more events

1. Unions of event E and event F, denoted by $E \cup F$ stands for the set of outcomes that are either in E or in F or in both. $E = \{H\}, F = \{T\}, E \cup F = \{H, T\}$, $E = (H, T), (H, H)\}$, $F = \{(H, T), (T, T)\}$, then $E \cup F = \{(H, T), (H, H), (T, T)\}$.
2. Intersections of event E and event F, denoted by $E \cup F$ are the set of outcomes that are in both E and F.
3. If events E and F have no element in common, we say that E and F are mutually exclusive and write $E \cap F = \phi$. Remember the ϕ stands for the null set.

4. Union and intersections of multiple events is denoted by $\cup_{i=1}^{n} E_i$ and $\cap_{i=1}^{n} E_i$.

5. E^c : Outcomes in S that are not in E.

6. E is contained in F, $E \subset F$: All of the outcomes of E that are also in F.

3 Refer to vein diagram in class

1. Commutativity: $A \cup B = B \cup A, A \cap B = B \cap A$

2. Associativity: $A \cup (B \cup C) = (A \cup B) \cup C, A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C), A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Display commutative, associative and distributive laws using the vein diagram.

4 de-Morgan's Laws

$(A \cup B)^c = A^c \cap B^c, (A \cap B)^c = A^c \cup B^c$.

5 Axioms of Probability

We introduce probability as a measure of frequency of occurrences.

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

How do we know that if the experiment is repeatedly performed over a second time, we should get a similar limiting proportion of heads? How about a set of simpler and more self-evident axioms? We assume that for each event E in the sample space S there exist a value $P(E)$, referred to as the probability of E. Recall the P is nothing but a mapping from the set of all events to $[0, 1]$.

1. Axiom 1:$0 \leq P(E) \leq 1$

2. Axiom 2: $P(S) = 1$.

3. For any sequence of mutually exclusive events $E_1, E_2, \ldots, E_k, \ldots$ if $E_i \cap E_j = \phi$ whenever $i \neq j$, we have

$$P(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$$

Examples:

1. Tossing a coin. If we assume that a head is as likely to appear as a tail, then we have $P(H) = P(T) = 1/2$ On the other hand, if the coin were biased and we felt that a head were twice as likely to appear as a tail, then we have $P(H) = 2/3$ and $P(T) = 1/3$
2. Rolling a die. If we suppose that all six sides are equally likely to appear, then we would have 1/6 for each possible number from 1 to 6. The probability of rolling an even number would be $P(2, 4, 6) = P(2) + P(4) + P(6) = 1/2$

5.1 Some simple propositions

1. $P(\phi) = 0$
2. For finitely many mutually exclusive events,

   $$P(\cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$$

   mention that when the sample space is finite Axiom 3 is equivalent to this one.
3. $P(E^c) = 1 - P(E)$
4. If $E \subset F$, then $P(E) \leq P(F)$. (probability of getting a 1 is less than prob of getting an odd number)
5. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Examples:

1. J is taking two books along on her holiday vacation. With probability .5 she will like the first book; with probability .4 she will like the second book; with probability .3 she will like both book. What is the probability she like neither book?