TEST #3 / STA 5326 (Distribution Theory) / Fall 2000

Directions

- This exam is **closed book** and **closed notes**.
- You have only two hours to do the exam.
- You will be given a copy of the Appendix (Table of Common Distributions) from the back of the text. You can use this on any problem except for those where you are explicitly told not to. You do NOT have to prove results from the Appendix.
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- **Begin each problem on a new page. Do NOT write on the backs of pages.** Staple the pages together in order when finished.
- Show and explain all your work (including your calculations). **No credit is given without work.** Partial credit is available. (If you know part of a solution – write it down. If you know an approach to a problem, but cannot carry it out – write down this approach.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).

Problem 1. Let X_1, X_2, X_3, X_4 be iid random variables with density $f(x) = e^{-x}$ for $x > 0$. Define $Y = \max_{1\leq i\leq 4} X_i$ and $Z = \min_{1\leq i\leq 4} X_i$.

(a) Find the cdf of $Z = \min X_i$.

(b) Find both the mean **and** variance of $Y = \max X_i$.

Problem 2. The random pair (X, Y) has the distribution:

		X = 1	2	3
	2	0	1/4	1/4
Y	3	1/6	0	1/6
	4	1/12	1/12	0

(a) Show that X and Y are **not** independent.

(b) Compute the value of $\text{Var}(Y \mid X = 2)$.

Problem 3. Show that the family of normal distributions (with **both** μ and σ^2 unknown) is an exponential family.

Problem 4. Compute EX^2 for the beta(α, β) distribution.

[Do **not** use the moments in the appendix. You may only use the appendix for looking up the pdf. Show and explain your work.]

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Problem 5. Let X and Y be iid with density $\frac{2}{\sqrt{2\pi}}e^{-z^2/2}$ for $z > 0$. (Note that X and Y are positive random variables.) Define $U = X^2 + Y^2$ and $W = X^2/(X^2 + Y^2)$.

(a) Find the joint density (pdf) of U and W. (Make sure to specify the support of this joint density.)

> **Hint:** Depending on how you arrange your algebra, you might find the following fact to be useful:
> $$\sqrt{\frac{z}{1-z}} + \sqrt{\frac{1-z}{z}} = \frac{1}{\sqrt{z(1-z)}}.$$

(b) Find $f_W(w)$, the marginal density of W.

> [If you failed to get the joint density in part (a), then (for partial credit) describe in detail how you would compute the marginal density of W if you were given the joint density.]

Problem 6. A packet of flower seeds contains $R + B$ seeds with R seeds of a red flowering variety and B seeds of a blue flowering variety. If a seed is planted (and properly cared for), it will produce flowers with probability p. Assume the seeds grow independently of each other.

Suppose that k seeds are selected at random from the packet and planted. Let X be the number of plants which produce red flowers. Find **both** the mean and variance of X.

> **Hint:** Introduce the random variable Y = the number of "red seeds" planted, and consider the conditional distribution of X given Y.