TEST #1
STA 5326
October 2, 2001

Name:_____

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- The exam has **6** pages and a total of **100 points**.

Problem 1. (13 pt) If $P(A^{\mathbf{C}}) = 8/13$ and $P(B^{\mathbf{C}}) = 3/13$, can A and B be disjoint? Explain. (Answer "yes" or "no" and give your explanation.)

Problem 2. (13 pt) Suppose X has pdf defined by $f(x) = 2x$ for $0 < x < 1$ and $f(x) = 0$ otherwise. Calculate Var $\left(X^{\mathbf{3}}\right)$.

Problem 3. Three players, A, B, and C, alternately and independently flip a coin and the first player to obtain a head wins. (Assume player A flips first so that the game proceeds $A, B, C, A, B, C, \ldots$.)

(a) (10 pt) Suppose that $P(\text{head}) = p$. What is the probability that A wins?

(b) (7 pt) Show that for all p, $0 < p < 1$, $P(A \text{ wins}) > 1/3$.

Problem 4. Consider the "Gambler's Ruin" problem with a biased coin having probability p of Heads (win a dollar) and $1-p$ of Tails (lose a dollar). Let $\psi(z)$ denote the probability of reaching a given goal of g dollars starting with an initial fortune of z dollars.

(a) (10 pt) Given that you reached the goal, what is the probability you lost the first **2** tosses? (Leave your answer in terms of ψ. Do NOT derive an expression for ψ.)

(b) (10 pt) Use the Law of Total Probability to find an equation that $\psi(z)$ must satisfy. (But do NOT derive an expression for ψ.)

Problem 5. Suppose X has density (pdf) $f_X(x) = \frac{2}{3}(x-1)$ for $\mathbf{2} < x < \mathbf{3}$.

(a) (10 pt) Find the pdf of $Y = -\log X$. (Make sure to specify the range of validity of your answer.)

(b) (9 pt) Find a monotone function $u(x)$ such that the random variable $Z = u(X)$ has a Uniform(0,1) distribution.

Problem 6. Suppose that b balls are placed at random into n cells.

(a) (9 pt) Suppose $b = n$. Find the probability that **exactly one** cell remains empty.

(b) (9 pt) Suppose $b > n$. Find a general formula for the probability that **every** cell contains at least one ball.