TEST #1
STA 5326 **Name:**_____
September 26, 2002

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages and a total of **100 points**.

Problem 1. For events A and B, find formulas for the probabilities of the following events in terms of the quantities $P(A)$, $P(B)$, and $P(A \cap B)$. Explain your reasoning.

(a) (9 pt) Either A or B but not both.

(b) (9 pt) At most one of A or B.

Problem 2. (12 pt) Ed is a contestant on a game show. He is presented with a bucket of n balls. The balls can be opened up; inside each ball is a check for some amount of money. Ed has no knowledge of the amounts written on the checks, but he is told that all the n amounts are different. Here are the rules of the game: Ed draws a sequence of balls at random (without replacement). After drawing each ball, Ed opens it and looks at the amount written on the check. He then must decide whether to keep the check and stop playing, or burn the check and draw another ball from the bucket.

Suppose Ed has drawn k balls. He has just read the amount on the k^{th} check and it is larger than those on the previous $k-1$ checks (which Ed has already burned). Given this, what is the probability that Ed has drawn the check worth the largest amount of money in the entire bucket?

Problem 3. (12 pt) Suppose $f(x) = ce^{-\mathbf{3}|x|}$, $-\infty < x < \infty$. Determine the value of c that makes $f(x)$ a density (pdf).

Problem 4. (12 pt) Let $X \sim \text{Poisson}\left(\frac{1}{2}\right)$. Calculate $E(X!)$, the expected value of "X factorial".

Problem 5. (14 pt) Suppose $Y = e^{(X-1)^2}$ where X has density (pdf) $f_X(x) = x^3/4$ for $0 < x < 2$. What is the density of Y?

Problem 6. A monkey types 6 digits at random. (Each keystroke is independent of the others with all 10 possibilities equally likely.)

(a) (10 pt) What is the probability the monkey types 123? (That is, the digits 123 occur as three consecutive digits somewhere in the six typed digits.)

Problem 6 continued: A monkey types 6 digits at random.

(b) (10 pt) What is the probability the monkey types 123 or 321?

Problem 7. (12 pt) A hat contains 10 coins; 3 coins are type A, and 7 are type B. The type A coins are biased with P(heads)=3/4, and the type B coins have P(heads)=1/4.

Choose a coin at random from the hat, toss it once, observe the result, and then put the coin back in the hat. Now do this again: choose another coin from the hat, toss it once and observe the result. Suppose you observed heads on both tosses. Given this, what is the probability that both the coins you drew were type A?