TEST #2
STA 5326 **Name:**______________________________________
October 31, 2002

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages and a total of **100 points**.

Problem 1. (16 pt) A prisoner has been sentenced to die within the next 30 days. He complains to the warden that he cannot bear to know in advance the exact moment of his death. The kindly warden agrees to help and gives the prisoner a bottle of 30 pills: one pill contains a swift poison, and the other 29 are harmless. Each day the prisoner swallows one pill chosen at random. Let X be the day on which the prisoner dies. (He takes the first pill on day 1, the second on day 2, etc.)

(a) What is $P(X = 25)$?

(b) What is EX?

Problem 2. (16 pt) Find the moment generating function (mgf) corresponding to the density (pdf) given by

$$f(x) = \frac{|x|}{c^2}, \quad -c < x < c\,.$$

Problem 3. (16 pt) The 10,000,000 inhabitants of a metropolitan area have all been exposed to varying amounts of a carcinogen, depending on their distance from the source. Suppose that we number the persons from 1 to 10,000,000 according to their distance from the source, and that person i has probability $\delta e^{-\beta i}$ of developing a rare form of cancer, where $\delta = .0002$ and $\beta = .0001$. What is the (approximate) probability that exactly **three** of these people will develop this form of cancer?

Problem 4. (16 pt) Let X be a continuous, nonnegative random variable [the pdf satisfies $f_X(x) = 0$ for $x < 0$]. Show that

$$EX = \int_0^\infty [1 - F_X(x)]\, dx$$

where $F_X(x)$ is the cdf of X.

Problem 5. (16 pt) Suppose the flow of traffic at a street corner is a sequence of Bernoulli trials with the probability of a car passing during any given second equal to p. A rather slow pedestrian can cross the street only if no car is to pass during the next **5** seconds. Find the probability that the pedestrian has to wait for exactly **6** seconds before starting to cross.

Problem 6. (16 pt) Prove that if X and Y are independent, $X \sim N(a, c)$, and $Y \sim N(b, d)$, then $X - Y \sim N(a - b, c + d)$. (Hint: Use mgf's)
[Notation: $N(a, c)$ means the normal distribution with $\mu = a$ and $\sigma^2 = c$, etc.]

[**No work is required on this problem.**]

Problem 7. (4 pt) Let T_r be the time until the r-th arrival in a Poisson process with rate λ. What is the distribution of T_r? (Give the name of the distribution and specify the values of any parameters in terms of r and λ. Use the notation in the appendix.)