TEST #3
STA 5326 **Name:**____________________________________
December 5, 2002

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- If an expression is valid only for a certain range of values, this range of values should be given as part of your answer. In particular, if a density or joint density is zero outside some region, this region should be stated as part of your answer.
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **12** pages and a total of **100 points**.

Problem 1. (14pt) Let (X, Y) be a bivariate random vector with joint pdf $f(x, y)$. (You may assume the support of (X, Y) is the entire plane R^2.) Let $U = aX + b$ and $V = cY + d$, where a, b, c, and d are fixed constants with $a > 0$ and $c > 0$. Find the joint pdf of (U, V). (Show your work.)

Problem 2. Let (X, Y) have the joint pdf

$$f(x,y) = \begin{cases} e^{-y} & \text{if } 0 < x < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Note: The support of this distribution is the part of the first quadrant which lies between the lines $x = 0$ and $x = y$.

(a) (5 pt) What is the marginal density function of X?

(b) (5 pt) What is the conditional density function $f_{Y|X}(y|x)$?

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[**Problem 2 continued**]

(c) (5 pt) What is $\mathrm{E}(Y|X = x)$?

Problem 3. Let $X \sim \text{Exponential}(\beta)$ and define $Y = X^{1/c}$ where $c > 0$ is a constant. Answer the following. (Show the work. Do NOT just quote results from the appendix.)

(a) (7 pt) Find the pdf of Y.

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[**Problem 3 continued**]

(b) (7 pt) Find the mean and variance of Y.

Problem 4. (14 pt) Let X and Y be independent random variables with $X \sim \text{Poisson}(\theta)$ and $Y \sim \text{Poisson}(\lambda)$. Show that $X \mid X + Y$ has a Binomial distribution.

Problem 5. Suppose that $X \sim \text{Exponential}(\beta)$ and $Y \mid X \sim \text{Uniform}(0, X)$.

(a) (5 pt) What is the joint pdf of (X, Y)?

(b) (10 pt) Find the pdf of $\frac{Y}{X+Y}$.

[**Extra Space for Work on Problem 5**] (just in case)

Problem 6. An urn contains R red balls, G green balls, and W white balls. A sample of k balls is drawn at random from this urn. Let X and Y be the number of red and green balls in the sample, respectively.

(a) (5 pt) Are X and Y independent? Answer "Yes" or "No" and justify your answer.
(No calculations are needed. If you wish, in this part you may assume the particular values $R = G = W = 10$ and $k = 4$.)

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[**Problem 6 continued**]

(b) (9 pt) Compute $E(XY)$.
(In this part, your answer should be general. Do NOT assume particular values for R, G, W, k.)

Problem 7. (14 pt) Show that the Gamma(α, β) distribution is unimodal. (For simplicity, assume $\alpha > 1$.)