TEST #1
STA 5326 **Name:**_____
September 30, 2003

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages and a total of **100 points**.

Problem 1. Approximately one-third of all human twins are identical (one-egg) and two thirds are fraternal (two-egg) twins. Identical twins are necessarily the same sex, with male and female being equally likely. Among fraternal twins, approximately one-fourth are both female, one-fourth are both male, and half are one male and one female. Finally, among all U.S. births, approximately 1 in 90 is a twin birth. Define the following events:

$$\begin{aligned} A &= \{\text{a U.S. birth results in twin } \textbf{males}\} \\ B &= \{\text{a U.S. birth results in } \textbf{fraternal} \text{ twins}\} \\ C &= \{\text{a U.S. birth results in twins}\} \end{aligned}$$

(a) (10 pt) Find $P(A \cap B \cap C)$.

(b) (8 pt) Choose a pair of twins at random. If **at least one** of the twins is a boy, what is the probability that both twins are boys?

Problem 2. (12 pt) Suppose that X has the Geometric pmf $f_X(x) = (1 - e^{-1})e^{-x}$ for $x = 0, 1, 2, \ldots$ Find the pmf of the random variable $Y = e^X$.

Problem 3. (18 pt) Suppose that X has density $f_X(x) = x^2/72$ for $0 \leq x \leq 6$ and

$$Y = \begin{cases} \frac{1}{3}X & \text{for } 0 \leq X < 3, \\ -\frac{1}{2}X + \frac{5}{2} & \text{for } 3 \leq X < 5, \\ X - 5 & \text{for } 5 \leq X \leq 6. \end{cases}$$

Find the density (pdf) of Y.

Problem 4. (16 pt) Let X have pdf $f_X(x) = \sqrt{\frac{2}{\pi}}\, e^{-x^2/2}$ for $0 < x < \infty$. Find the variance of X.

Problem 5. There are c contestants who enter a grueling bicycle race consisting of s stages. All the stages are equally difficult and all the contestants have equal ability. The contestants and stages are independent of each other. A contestant will finish any stage with probability p. If a contestant fails to complete any stage, he/she is out of the race.

(a) (8 pt) What is the probability that at least one contestant finishes the race?

(b) (8 pt) What is the probability that exactly one contestant finishes the race? (That is, exactly one contestant finishes all s stages.)

[**Problem 5 continued**]

(c) (4 pt) What is the probability that at least one candidate starts the final stage (stage s), but nobody finishes the race?
(Note: 'but' and 'and' are logically equivalent.)

Problem 6. (16 pt) There are n married couples at a party. Prizes are given to k people (where $k < n$) selected as follows. All the $2n$ names of the party goers are placed in a hat, and k names are selected at random (withOUT replacement). What is the probability that there is **at least one** married couple in which both the husband and wife receive a prize?