TEST #3
STA 5326 **Name:**____________________________________
December 4, 2003

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Different parts of the same problem are often unrelated and can be solved independently of each other. If you cannot do one part of a problem, you should still go on and try to do the later parts.
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages and a total of **100 points**.

Problem 1. Suppose X has a Beta(2,1) distribution with density $f_X(x) = 2x$ for $0 < x < 1$, and given X, the random variable Y has a binomial distribution with n trials and success probability equal to X.

(a) (6 pt) Find EY.

(b) (6 pt) Find $\mathrm{Var}(Y)$.

(c) (3 pt) Find $E(XY \mid X)$.

(d) (6 pt) Find the joint distribution of X and Y.
(Your answer should include the support of (X, Y).)

(e) (6 pt) Find the marginal distribution of Y

Problem 2. Consider the pdf

$$f(x) = \begin{cases} e^x & \text{for } 0 < x < \ln 2 \, (\approx .693) \\ 0 & \text{otherwise.} \end{cases}$$

(a) (14pt) Graph $(1/\sigma) f((x-\mu)/\sigma)$ for each of the following on the same axes.
(a) $\mu = 0, \sigma = 1$
(b) $\mu = 3, \sigma = 1$
(c) $\mu = 3, \sigma = 2$
Please draw a careful plot, marking the important points on the axes. Label the curves (a),(b),(c).

(b) (6 pt) With $f(x)$ as given above, does the family of densities $(1/\sigma) f((x-\mu)/\sigma)$ for $-\infty < \mu < \infty$ and $0 < \sigma < \infty$ form an exponential family? (Answer "Yes" or "No" and justify your answer.)

Problem 3. Let X and Y be independent random variables with $X \sim \text{gamma}(r, 1)$ and $Y \sim \text{gamma}(s, 1)$. Define $U = X + Y$ and $Z = Y/X$.

(a) (24 pt) Find the joint density of (U, Z).
(Your answer should include the support of (U, Z).)

[**Problem 3 continued**]

(b) (5 pt) Show that U and Z are independent.

Problem 4. Suppose (X, Y) have a bivariate normal distribution with $\mu_X = \mu_Y = 0$, $\sigma_X^2 = \sigma_Y^2 = 1$ and $\rho = 1/\sqrt{2}$ with density

$$\frac{1}{\pi\sqrt{2}} \exp\left(-x^2 - y^2 + \sqrt{2}\,xy\right)$$

for $-\infty < x < \infty$, $-\infty < y < \infty$.

(a) (16 pt) Derive $f_{Y|X}(y|x)$, the conditional density of Y given $X = x$.
(Calculate this from the joint density, **NOT** by quoting facts about the bivariate normal distribution.)

(b) (8 pt) Let a and b be constants. What is the distribution of $Z = aX + bY$? State the name of the distribution and the values of any parameters.
(You may do this one by quoting facts about the bivariate normal distribution. You do **NOT** have to work with the joint density.)