TEST #2
STA 5326 **Name:**____________________________________
October 28, 2004

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations). **No credit is given without work.** But don't get carried away! Show just enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Unless otherwise stated, arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **7** pages and a total of **100 points**.

Problem 1. (18 pt) Use indicator random variables to prove the following:

$$P(A \cap B^c \cap C^c) = P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

Problem 2. (18 pt) Suppose that 10,000,000 voters cast their votes for candidates A and B by flipping a fair coin. (If the coin comes up heads, they vote for A; if tails, for B.) What is the probability that the two candidates will be within 517 votes of each other? **(Give an approximate numerical answer.)**

Problem 3. (18 pt) Suppose X is a discrete random variable with pmf

$$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{for } x = 0, 1, 2, 3, \ldots$$

Compute $M_X(t)$, the mgf of X. (Do NOT use the appendix.)

Problem 4. Consider a sequence of independent coin flips, each of which has probability p of being heads. Define a random variable X as the length of the run (of either heads or tails) started by the first trial. (For example, $X = 3$ if either $TTTH$ or $HHHT$ is observed.)

(a) (9 pt) Find $P(X \geq k)$ for $k = 1, 2, 3, \ldots$

(b) (9 pt) Find EX.

Problem 5. (18 pt) The hazard function $h_T(t)$ associated with the random variable T is defined by

$$h_T(t) = \lim_{\delta \downarrow 0} \frac{1}{\delta} P(t \leq T < t + \delta \mid T \geq t) \, .$$

Show that if T is a continuous random variable, then

$$h_T(t) = \frac{f_T(t)}{1 - F_T(t)} \, .$$

Quickies: No work is required for the following. Just state the answers. These are worth 2 points each.

If the answer is a distribution, make sure to give both the name of the distribution **and the values of any parameters.** Use the notation in the appendix.

Problem 6. (2 pt) If T has an exponential distribution with parameter β, what is the hazard function $h_T(t)$? (See the previous problem for a definition.)

Problem 7. (2 pt) For what values of t is the integral given below finite?

$$\int_{-\infty}^{\infty} \frac{e^{tx}}{\pi(1+x^2)}\,dx$$

Problem 8. (2 pt) If $X_n \sim \text{Geometric}(p = 1/n)$, then we know there is a random variable Y such that

$$\frac{X_n}{n} \xrightarrow{d} Y \qquad \text{as } n \to \infty\,.$$

What is the distribution of Y?
(The symbol $\xrightarrow{d}$ means "converges in distribution".)

Problem 9. (2 pt) If $X_n \sim \text{Poisson}(\lambda = n)$, then we know there is a random variable Y such that

$$\frac{X_n - n}{\sqrt{n}} \xrightarrow{d} Y \qquad \text{as } n \to \infty\,.$$

What is the distribution of Y?

Problem 10. (2 pt) If X_1, X_2, X_3, X_4, X_5 are iid Exponential($\beta = 8$) random variables, then what is the distribution of $\min\limits_{1\le i\le 5} X_i$?