TEST #1
STA 5326 **Name:**_____
September 29, 2005

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations) for all the problems except those on the last page. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages and a total of **100 points**.

Problem 1. (14 pt) Two pennies, one with $P(\text{head}) = \alpha$ and one with $P(\text{head}) = \beta$, are to be tossed together independently. Define $p_i = P(i \text{ heads occur})$ for $i = 0, 1, 2$. Can α and β be chosen so that $8p_0 = p_1 = 8p_2$? Prove your answer.

Problem 2. (14 pt) Suppose that X has density (pdf) $f_X(x) = 2x$ for $0 < x < 1$. Define

$$Y = \begin{cases} 2 & \text{for } X < 1/3 \\ 2 + 6X & \text{for } 1/3 \leq X < 2/3 \\ 9 & \text{for } X \geq 2/3 \end{cases}$$

Find the cdf of Y. (Make sure you specify $F_Y(y)$ for all real values y.)

Problem 3. Suppose X has pdf $f(x)$ defined by $f(x) = \frac{1}{2}(1+x)^{-2}$ if $x \geq 0$ and $f(x) = \frac{1}{2}(1-x)^{-2}$ if $x < 0$.

(a) (8 pt) Find $P(X < t)$ for all t. Evaluate all integrals.

(b) (6 pt) Find $P(|X| < t)$ for all t. Evaluate all integrals.

Problem 4. (14 pt) Two people (A and B) each toss a fair coin n times. Find the probability that A tosses exactly one more head than B does. (Your answer may be left as a summation.)

Problem 5. (14 pt) Suppose you have a fair coin with the sides labeled $+1$ and -1. Toss this coin 3 times and let X_i be the value observed on the i-th toss. Define $X_4 = X_1X_2X_3$. For $i = 1, 2, 3, 4$, define A_i to be the event that $X_i = 1$.

Show that the events A_1, A_2, A_3, A_4 are *not* mutually independent.

Problem 6. (14 pt) A children's card game (Barnyard Poker) has a deck of 48 cards. Each card displays one of 8 different animals. There are 6 chickens, 6 cows, 6 pigs, 6 dogs, 6 sheep, 6 horses, 6 geese, and 6 cats. Players are dealt a "hand" of 5 cards from a well shuffled deck. What is the probability of drawing a full house? (A full house is three animals of one kind and two of another, for example, 3 chickens and 2 cows.)

Problem 7. (8 pt) Let A, B, C, D be arbitrary events. Use the principle of inclusion-exclusion to give an expression for $P(A \cup B \cup C \cup D)$. (Write out all the terms explicitly without using ... or summation signs. No work is required. Do **NOT** give a proof.)

$P(A \cup B \cup C \cup D) =$

Problem 8. (8 pt) Suppose X has cumulative distribution function (cdf) $F(x) = x^8/\pi^8$ for $0 \leq x \leq \pi$. Let $Y = \sin X$.

Find $P(Y > c)$ where $0 < c < 1$. (State the answer and draw an appropriate picture, labeling relevant points on the axes. No other work is required.)