TEST #2
STA 5326 **Name:**____________________________________
November 3, 2005

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations) for all the problems unless you are told otherwise. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.

  When doing algebra and calculus, show enough detail so that the grader can see in their head without difficulty how to get from each line to the next.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages and a total of **100 points**.

Problem 1. (16 pt) At a carnival game customers buy a bucket of balls and for every ball they succeed in tossing through a hoop they get a crummy little prize. To compensate for supposed differences in skill, men are given a bucket of 12 balls, women a bucket of 15 balls, and children a bucket of 18 balls. But in fact there is absolutely no skill involved in the game; every ball goes through the hoop with probability 1/2 independently of the other balls.

In a particular week, 1000 men, 2000 women, and 4000 children play the game. During this week, what is the probability that there is exactly one person who doesn't win any prizes. (Compute a numerical answer. You may use an appropriate approximation.)

Problem 2. (12 pt) Find the median of the distribution with density $f(x) = 3e^{-3x}$ for $x > 0$.

Problem 3. Suppose X has pdf $f(x)$ which is symmetric about the point a, that is,

$$f(a+u) = f(a-u) \quad \text{for all } u.$$

(a) (8 pt) Prove that the median of X is a.

[**Problem 3 continued**]

Suppose X has pdf $f(x)$ which is symmetric about the point a, that is,

$$f(a+u) = f(a-u) \quad \text{for all } u.$$

(b) (8 pt) Suppose EX exists. Prove that $EX = a$.

Problem 4. Let the random variable X have the pdf $f(x) = 2xe^{-x^2}$ for $x > 0$.

(a) (6 pt) Find the mean of X

(b) (6 pt) Find the variance of X.

Let the random variable X have the pdf $f(x) = 2xe^{-x^2}$ for $x > 0$.

(c) (8 pt) Find the transformation $g(X) = Y$ and values α and β so that $Y \sim \text{gamma}(\alpha, \beta)$.

Problem 5. (16 pt) Al and Bob each toss a fair coin 1000 times. What is the probability they get exactly the same number of heads? (Compute a numerical answer using an appropriate approximation.)

Problem 6. Suppose X_1 and X_2 are independent with $X_1 \sim \text{Gamma}(\alpha_1, \beta_1)$ and $X_2 \sim \text{Gamma}(\alpha_2, \beta_2)$.

(a) (2 pt) Under what condition(s) on α_1, α_2, β_1, β_2 does $X_1 + X_2$ have a Gamma distribution?

(b) (6 pt) Prove the "closure" property you describe in part (a).

Problem 7. Define $M(t) = \dfrac{3}{1-3t}$ for $t < 1/3$.

(a) (2 pt) Is this the moment generating function of some distribution? (Answer Yes or No.)

(b) (4 pt) If it is an mgf, state for what distribution. If it is **not** an mgf, state a reason why it cannot be one.

Problem 8. (6 pt) Let X be the time of the k^{th} arrival in a Poisson process with rate λ. What is the distribution of X?
(Give the name of the distribution and the values of any parameters. **No work is required.**)