**TEST #3**
**STA 5326** **Name:**__________________________________
**December 8, 2005**

**Please read the following directions.**
**DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO**

# Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations) for all the problems unless you are told otherwise. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.

  When doing algebra and calculus, show enough detail so that the grader can see in their head without difficulty how to get from each line to the next.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **7** pages and a total of **100 points**.

**Problem 1.** Suppose

$$X \sim \text{Exponential}(\beta) \quad \text{and} \quad Y \mid X \sim \text{Poisson}(\theta X)$$

where $\theta$ and $\beta$ are positive.

**(a)** (12 pt) Find the variance of $Y$.

**(b)** (6 pt) Find $E[\,(Y - \theta X)\log X\,]$.

[**Problem 1 continued**]
Continue supposing

$$X \sim \text{Exponential}(\beta) \quad \text{and} \quad Y \mid X \sim \text{Poisson}(\theta X)\,.$$

**(c)** (12 pt) Find the marginal distribution of $Y$.

**Problem 2.** (16 pt) Suppose $X$ and $Y$ have the joint density

$$f(x,y) = \begin{cases} 6y & \text{for } (x,y) \in D \\ 0 & \text{otherwise} \end{cases}$$

where $D$ is the triangular region where $x > 0$, $y > 0$, and $x + y < 1$.

Find $P\left\{ Y \geq \frac{1}{2}(1 - X) \right\}$.

**Problem 3.** Suppose $X$ and $Y$ are independent with $X \sim N(0,1)$ and $Y \sim \text{Exponential}(\beta = 2)$.

**(a)** (20 pt) Find the joint density of $S$ and $U$ where

$$S = X^2 + Y \quad \text{and} \quad U = \frac{Y}{X^2 + Y}.$$

(Your answer should include the support of the distribution.)

**(b)** (4 pt) Are $S$ and $U$ independent? Answer "Yes" or "No" and justify your answer.

**Problem 4.**

**(a)** (6 pt) State the definition of a general $k$ parameter exponential family.

**(b)** (10 pt) Show that the family of normal distributions with both $\mu$ and $\sigma$ unknown is a two parameter exponential family.

**Problem 5.** (10 pt) Let $X_1$, $X_2$, $X_3$ be uncorrelated random variables, each with mean $\mu$ and variance $\sigma^2$. Find, in terms of $\mu$ and $\sigma^2$, an expression for $\mathrm{Cov}(X_1 - X_3, X_2 + X_3)$.

**Problem 6.** (4 pt) Suppose $X$ and $Y$ have joint cumulative distribution function (cdf) $F(x, y)$. Then it will be true that

$$P(a < X \leq b,\ c < Y \leq d) = \pm F(b, d) \pm F(b, c) \pm F(a, d) \pm F(a, c)$$

if the appropriate sign (plus or minus) is chosen for each term. Write the correct sign in each of the boxes below. (**No work is required,** but room for work is given in case you need it.)

$$P(a < X \leq b,\ c < Y \leq d) = \square\, F(b, d) \,\square\, F(b, c) \,\square\, F(a, d) \,\square\, F(a, c)$$