TEST #3
STA 5326 **Name:**_____
December 7, 2006

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations) on all the problems. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- If you need more room to work a problem, use the back of the same page and write "work on back" to indicate that you have done so.
- The exam has **8** pages and a total of **100 points**.

Problem 1. Suppose X and Y have the joint pdf

$$f(x,y) = \begin{cases} \frac{1}{5}(3x+y) & \text{if } 0 < y < 2 \text{ and } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) (10 pt) Find the marginal density of Y.

(b) (10 pt) Find $E(X \mid Y = y)$.

[**Problem 1 continued**]
We continue using the joint pdf

$$f(x,y) = \begin{cases} \frac{1}{5}(3x+y) & \text{if } 0 < y < 2 \text{ and } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(c) (10 pt) Find the joint **c**df $F_{X,Y}(x,y)$ of X and Y.
(Give the answer for all possible values of x and y.)

[**Problem 1 continued**]
We continue using the joint pdf

$$f(x,y) = \begin{cases} \frac{1}{5}(3x+y) & \text{if } 0 < y < 2 \text{ and } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(d) (10 pt) Let $U = 2X - 1$ and $W = 3Y + 2$. Find the joint pdf of (U, W). (Prove your answer. Make sure to specify the support of the joint pdf.)

Problem 2. Suppose the distribution of Y, conditional on $X = x$, is $N(3x, 7)$ and that the marginal distribution of X is $N(2, 5)$.

(a) (8 pt) Find EY.

(b) (8 pt) Find $\text{Var}(Y)$.

(c) (8 pt) Let $U = X^4$ and $W = Y - 3X$. Find $\text{Cov}(U, W)$.

Problem 3. Suppose X_1, X_2, X_3 are iid with pdf $f(x)$ and cdf F. Let $(U_1, U_2, U_3) = (X_{(1)}, X_{(2)}, X_{(3)})$ be the order statistics.

(a) (4 pt) What is the joint pdf of (U_1, U_2, U_3)? (No work required. Just state the answer.)

(b) (6 pt) Use the answer to part (a) to calculate the density of $U_2 = X_{(2)}$.

Problem 4. (14 pt) Suppose X and Y are independent random variables with pdf's $f_X(x)$ and $f_Y(y)$, respectively. Derive a general formula for the density of $Z = XY$.

Problem 5. (6 pt) Let $X \sim N(0, \sigma^2)$. Define $Y = 3X - 2$ and $Z = 5X + 7$. Calculate $\mathrm{Cov}(Y, Z)$.

Problem 6. (6 pt) Suppose X and Y have pdf

$$f_{X,Y}(x, y) = \begin{cases} ye^{-x} & \text{for } 0 < y < x < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent? Answer "Yes" or "No" and justify your choice. (No calculations are required.)