TEST #1
STA 5326 **Name:**_____
September 25, 2008

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- The different parts of a problem are sometimes unrelated. If you cannot solve part of a problem, you should still go on to look at the later parts.
- If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.
- Show and explain your work (including your calculations) for all the problems except Problem 6. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages.
- There are a total of **104** points.

Problem 1. A dice game is played as follows. A bucket of dice contains 7 red dice, 7 green dice, and 7 blue dice, for a total of 21 dice. All of the dice are fair and 6-sided. The bucket is well mixed and a player, with his eyes closed, reaches in and grabs **4** dice. All 4 dice are simultaneously thrown. A throw of the dice receives points if (*i*) any numbers are repeated, (*ii*) all 4 dice are the same color, or (*iii*) the numbers can be arranged in a consecutive sequence (for example: 2,3,4,5). For a single throw of the dice, compute the following.

(a) (7%) What is the probability there are **no** repeated numbers?

(b) (7%) What is the probability all 4 dice are the same color?

[**Problem 1 continued**]

(c) (7%) What is the probability the numbers can be arranged in a consecutive sequence?

(d) (7%) What is the probability the throw is completely worthless (receives zero points)?

Problem 2. (15%) Find the pdf of Y when $Y = e^{-X}$ and X has the pdf

$$f_X(x) = 30x^2(1-x)^2\,, \quad 0 < x < 1.$$

Problem 3. (15%) Find the pdf of Y when X has pdf $f_X(x) = e^{-x}$ for $x > 0$, and

$$Y = \begin{cases} 2X & \text{for } X \leq 4 \\ 12 - X & \text{for } 4 < X \leq 7 \\ 3X - 16 & \text{for } X > 7\,. \end{cases}$$

Problem 4. There are 80 students at a fraternity party, with equal numbers of freshmen, sophomores, juniors, and seniors (20 of each). Eight people are randomly selected to receive door prizes.

(a) (8%) What is the probability these eight prizes are equally distributed to the four classes (two to each)?

(b) (8%) What is the probability that **no** freshmen receive prizes?

Problem 5. In a population, the proportion of good and bad drivers is 2/3 and 1/3, respectively. In any given year, a good driver will have an accident with probability 1/4, and a bad driver with probability 3/4. A driver is selected at random from this population and observed for 3 years.

(a) (8%) What is the probability the driver has accidents in exactly 2 of these 3 years?

(b) (8%) If the driver has accidents in exactly 2 of the 3 years, what is the probability the driver is bad?

Problem 6. (10%) The random variable X has cdf

$$F(x) = \begin{cases} 0 & \text{for } x \;?\; 0 \\ x + 0.10 & \text{for } 0 \;?\; x \;?\; 0.3 \\ x + 0.25 & \text{for } 0.3 \;?\; x \;?\; 0.6 \\ 1 & \text{for } 0.6 \;?\; x \end{cases}$$

where each question mark (?) stands for either $<$ or $\leq$ (but I'm not telling you which).
Find values for the following. (No work is required. Just fill in the blanks. They are worth 2% each.)

$\boldsymbol{F(0.3) =}$ _____

$\boldsymbol{P(0 < X < 0.6) =}$ _____

$\boldsymbol{P(X = 0.3) =}$ _____

$\boldsymbol{P(X \geq 1) =}$ _____

$\boldsymbol{P(X = 0.5) =}$ _____

If you have finished everything else and have checked your answers, then try this. Partial credit will be minimal. Your solution must be nearly correct to receive any credit.

Problem 7. (4%) A three-legged alien has a closet containing n triplets of shoes. If $3k$ shoes are chosen at random ($k < n$), what is the probability that there will be no matching triplet in the sample?