TEST #2
STA 5326
October 29, 2009

Name:__________________________________

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- The different parts of a problem are sometimes unrelated. If you cannot solve part of a problem, you should still go on to look at the later parts.
- If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.
- Show and explain your work (including your calculations) for all the problems except those on the last page. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- On problems where work is shown, **circle your answer**. (On these problems, you should give only one answer!)
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely **unless numerical answers are requested**. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages.
- There are a total of **100** points.

Problem 1. (15%) Suppose that X is a continuous random variable with median m, that is, $P(X \leq m) = P(X \geq m) = 1/2$. Show that $E|X - a|$ is minimized when $a = m$.

Problem 2. An urn initially contains two balls, one red and one green. This is a Polya urn: after a ball is drawn, it is replaced and then another ball of the same color is added to the urn. Ed decides to randomly draw balls until he finally gets a green ball. Then he will stop. Let X be the total number of balls that Ed draws from the urn.

(a) (8%) For $k = 1, 2, 3, \ldots$, what is $P(X = k)$?

(b) (7%) What is EX?

Problem 3. Let X have the density (pdf)

$$f(x) = cx^3 e^{-x^2/\beta^2}, \quad 0 < x < \infty, \quad \beta > 0.$$

(a) (7%) Find the value of the normalizing constant c. (c will depend on β.)

(b) (8%) Find EX.
[If you could not find the value of c, just leave it as c in your answer.]

Problem 4. Suppose X and Y are iid random variables with moment generating function

$$M_X(t) = M_Y(t) = \frac{1}{(1+t)(1-t)} \quad \text{for } -1 < t < 1 \text{ (and undefined or } +\infty \text{ otherwise).}$$

Find the following moment generating functions. In each case, your answer should include the range in which the mgf is well-defined and finite.

(a) (8%) Find the mgf of $3X + 5$.

(b) (7%) Find the mgf of $X + Y$.

Problem 5. (16%) Suppose X is a random variable with moment generating function (mgf) given by

$$M(t) = e^{\beta(e^{\lambda t} - 1)} = \exp[\beta(e^{\lambda t} - 1)] \qquad \text{where } \beta > 0.$$

Use the mgf to find the mean and variance of X.

mean=__________________________ variance=__________________________

Problem 6. Let $X \sim$ Negative Binomial(r, p) according to the textbook definition (in which the possible values are $0, 1, 2, \ldots$). See the appendix for the pmf, mean, variance, and mgf.

In the following parts, compute a **numerical** answer using an appropriate **approximation**. Briefly justify the approximation that you use.

In each part, let $\mu = EX$ and $\sigma^2 = \text{Var}(X)$.

(a) (9%) Suppose $r = \mathbf{10^{10}}$ and $p = \mathbf{.25}$. Find $P\left(\mu - \frac{\sigma}{2} < X < \mu + \frac{\sigma}{2}\right)$.

[**Problem 6 continued**]

(b) (6%) Suppose $r = \mathbf{10^{10}}$ and $p = \mathbf{1 - 10^{-10}}$. Find $P\left(\mu - \frac{\sigma}{2} < X < \mu + \frac{\sigma}{2}\right)$.

The remaining problems are multiple choice. Circle the single correct response. No work is required.

Problem 7. (3%) In the neighborhood of $t = 0$, which one of the following is the moment generating function (mgf) of some distribution?

a) $\frac{t}{1-2t}$ **b)** $\frac{2t}{1+2t}$ **c)** $\frac{2}{1-2t}$ **d)** $\frac{1}{t(1-2t)}$ **e)** $\frac{2}{t(1-t)}$ **f)** $\frac{1}{1-2t}$ **g)** $\frac{2t}{1-t}$

Problem 8. (3%) Suppose the possible values of a random variable X are $0, 1, 2, \ldots, 9, 10$. For this random variable, what is the set of values of t for which the moment generating function (mgf) $M(t)$ is well-defined and finite?

a) $[0, \infty)$ **b)** $(-\infty, 0]$ **c)** $(0, \infty)$ **d)** $(-\infty, 0)$ **e)** $(-\infty, \infty)$ **f)** $[0, 10]$ **g)** $(0, 10)$

h) $[10, \infty)$ **i)** $(10, \infty)$ **j)** not enough information is given to determine the answer

Problem 9. (3%) Suppose X is a random variable, and y and z are positive values. Then

$$P(X > y \,|\, X > y + z)$$

is equal to which of the following?

a) 0 **b)** 1 **c)** 1/2 **d)** $\frac{P(y < X < y+z)}{P(X > y+z)}$ **e)** $\frac{P(y < X < y+z)}{P(X > y)}$

f) $\frac{P(X > y)}{P(y < X < y+z)}$ **g)** $\frac{P(X > y+z)}{P(y < X < y+z)}$ **h)** $\frac{P(X > y+z)}{P(X > y)}$ **i)** $\frac{P(X > y)}{P(X > y+z)}$