TEST #3
STA 5326 **Name:**____________________________________
December 6, 2012

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Show and explain your work (including your calculations) for all problems unless you are explicitly told otherwise. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- The different problems are not related. The different parts of a problem are sometimes unrelated. If you cannot solve part of a problem, you should still go on to look at the later parts.
- On problems where work is shown, **circle your answer**. (On these problems, you should give only one answer!)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages.
- There are a total of **100** points.

Problem 1. **Two** white balls and **three** black balls are distributed in two urns with **two** balls in the first urn and **three** in the second. We say the system is in state i, $i = 0, 1, 2$, if the first urn contains i white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn.

(a) (10%) Calculate the transition probability matrix for this Markov chain.

[**Problem 1 continued**]

(b) (10%) Find the stationary distribution of this chain.
(If you did not answer the previous part, just call the transition probabilities P_{ij} and give a detailed description of the process you would use to find the stationary distribution.)

Problem 2. Suppose X and Y are independent with densities

$$f_X(x) = 2x \quad \text{for } 0 < x < 1, \qquad f_Y(y) = \frac{y^4 e^{-y}}{4!} \quad \text{for } 0 < y < \infty\,.$$

Define $U = Y\sqrt{X}$ and $V = Y$.

(a) (14%) Find the joint density of (U, V). (Make sure to specify the support.)

[**Problem 2 continued**]

(b) (6%) Find the marginal density of U. (Specify the support.)

(c) (4%) Are U and V independent? Answer "Yes" or "No" and justify your answer.

Problem 3. (12%) Let $X \sim \text{Poisson}(\alpha)$ and $Y \sim \text{Poisson}(\beta)$, with X and Y independent. Calculate $P(X = k | X + Y = n)$ where $0 \leq k \leq n$.

Problem 4. Suppose $X|P \sim$ Negative Binomial(r, P) and $P \sim$ Beta(α, β) where $\alpha > 2$. (In your answers below, you do NOT have to simplify the expressions involving gamma functions.)

(a) (10%) Find EX.

(b) (10%) Find $\text{Var}(X)$.

Problem 5. Suppose $N(\cdot)$ is a non-homogeneous Poisson process with rate function $\lambda(t) = t^2$.

(a) (6%) What is the distribution of $N(2) - N(1)$? (Give the name of the distribution and specify the value(s) of any parameter(s).)

(b) (4%) What is the approximate probability of an arrival in the time interval $(3, 3 + 10^{-6})$?

Problem 6. Suppose $\text{Var}(X) = \text{Var}(Y) = \alpha$ and $\text{Cov}(X, Y) = \beta$.

(a) (4%) Find the correlation ρ_{UV} between $U = 2X + 3$ and $V = 5Y + 7$.

(b) (4%) Find $\text{Cov}(X + Y, X - Y)$.

Problem 7. (6%) Suppose X and Y are discrete random variables whose joint pmf is described by the following table:

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	1/4	1/6	1/12
$Y = 1$	1/12	1/12	1/12
$Y = 2$	1/24	1/12	1/8

Find $\text{Var}(X|Y = 0)$.