TEST #2
STA 5326 **Name:**____________________________________
October 31, 2013

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.
- Show and explain your work (including your calculations) for all problems unless you are explicitly told otherwise. **No credit is given without work.** But don't get carried away! Show enough work so that what you have done is clearly understandable.
- On problems where work is shown, **circle your answer**. (On these problems, you should give only one answer!)
- All the work on the exam should be your own. No "cooperation" is allowed.
- Arithmetic does **not** have to be done completely, unless stated otherwise. Answers can be left as fractions or products. You do not have to evaluate binomial coefficients, factorials or large powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need only pens, pencils, erasers and a calculator. (You will be supplied with scratch paper.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **10** pages.
- There are a total of **100** points.

Problem 1. (13%) For $0 < p < 1$ and any integer $n \geq 3$, show that

$$\sum_{k=0}^{2} \binom{n}{k} p^k (1-p)^{n-k} = \frac{n(n-1)(n-2)}{2} \int_0^{1-p} t^{n-3}(1-t)^2 \, dt \,.$$

[**Problem 1 continued**] (Additional work space)

Problem 2. Consider the Pareto pdf defined by

$$f(x) = \frac{\beta\alpha^{\beta}}{x^{\beta+1}}\,, \qquad \alpha < x < \infty,\ \ \alpha > 0,\ \ \beta > 0\,.$$

(a) (6%) Verify that $f(x)$ is a pdf.

(b) (6%) Compute the **mean** of this distribution, and specify the values of β for which the mean exists.

[**Problem 2 continued**]

(c) (6%) Compute the **variance** of this distribution, and specify the values of β for which the variance exists.

Problem 3.

(a) (6%) For a nonnegative random variable T, a formula expressing the hazard function $h(t)$ in terms of the pdf f and cdf F was stated in lecture and proved in the homework. What is this formula? (Just give the formula. No work is required.)

(b) (6%) If T has the Pareto pdf used in the previous problem

$$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \qquad \alpha < x < \infty, \;\; \alpha > 0, \;\; \beta > 0,$$

what is the hazard function of T?

(c) (6%) Suppose a creature has lifetime T having the pdf given above with $\alpha = 2$ and $\beta = 3$. If the creature is alive at time 4.000, what is the approximate probability it will die before time 4.001 ?

Problem 4. Suppose $X_1, X_2, \ldots, X_n$ are iid with the Pareto pdf used in the previous problems:

$$f(x) = \frac{\beta\alpha^{\beta}}{x^{\beta+1}}, \qquad \alpha < x < \infty, \;\; \alpha > 0, \;\; \beta > 0\,.$$

Define $Z = \min X_i$.

(a) (8%) Find the cdf of Z.

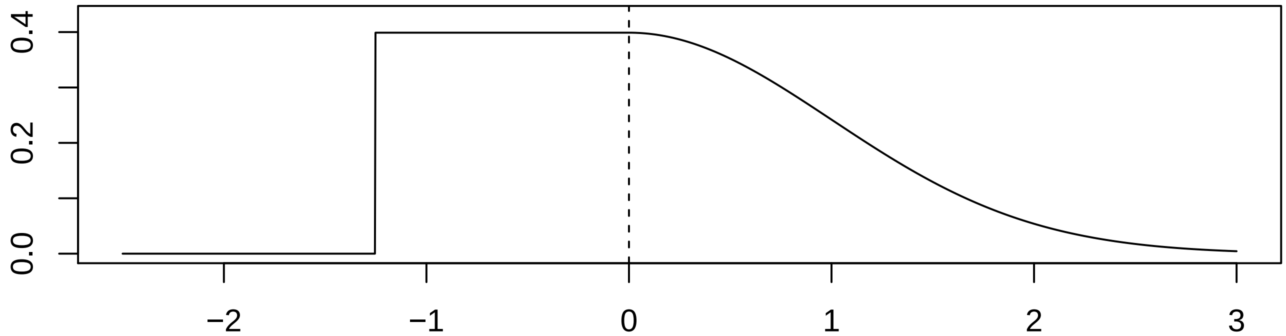
(b) (7%) Find EZ

Problem 5. (15%) Suppose $X \sim \text{Beta}(\alpha = 200, \beta = 600)$. Calculate a normal approximation for $P(.254 < X < .280)$. (Your final answer should be a numerical value.)

Problem 6. (12%) Show that if $X \sim \text{Geometric}(p)$, then

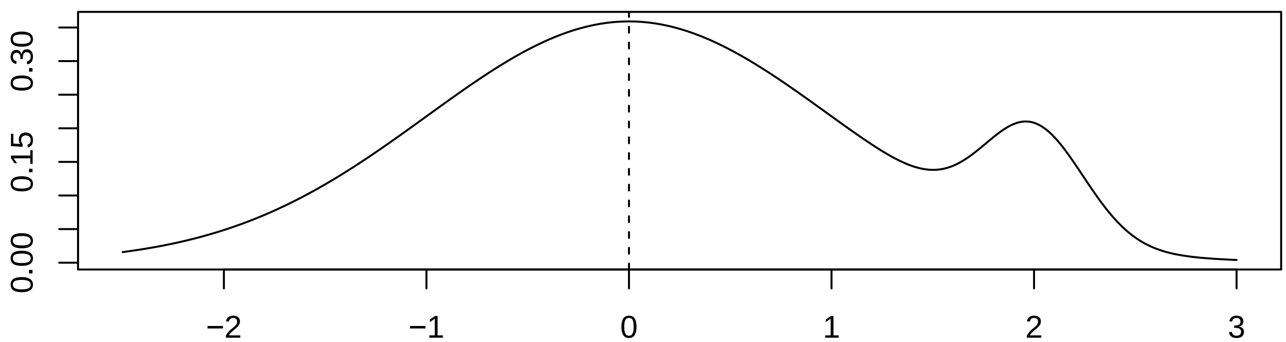
$$P(X > y + z \,|\, X > y) = P(X > z) \quad \text{for all integers } y, z > 0.$$

The term “unimodal” is sometimes used loosely, but a careful definition is given by the following: A pdf is said to be “unimodal with mode a” if $f(a) \geq f(x) \geq f(y)$ whenever $a \leq x \leq y$ or $a \geq x \geq y$. Use this definition to answer the following two questions. (Circle the correct response.) Assume that all pdf's given below are zero outside of the range pictured.



Problem 7. (3%) The pdf pictured **above** is unimodal with mode 0.

a) True **b**) False **c**) Cannot be determined from the given information



Problem 8. (3%) The pdf pictured **above** is unimodal with mode 0.

a) True **b**) False **c**) Cannot be determined from the given information

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Problem 9. (3%) Suppose X is a random variable with mean μ. The formula for the standardized **kurtosis** of X contains which one of the following quantities.

a) Ee^{tX} **b**) $\Gamma(X)$ **c**) $\Gamma(\alpha)$ **d**) $\dfrac{\Gamma(\alpha+3)}{\Gamma(\alpha)}$ **e**) $M_X^{(3)}(0)$

f) $\binom{n}{X}$ **g**) $f_X(x)$ **h**) $F_X(x)$ **i**) $E(X-\mu)^4$ **j**) $E(X-\mu)^3$