

Please read the following directions.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the “Table of Common Distributions” given in the back of the text.)
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- **If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.**
- Show and explain your work (including your calculations) for all problems except those on the last page. **No credit is given without work.** But don’t get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- All the work on the exam should be your own. No “cooperation” is allowed.
- Simplify your answers when it is easy to do so. But more difficult arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate large binomial coefficients, factorials or powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need to bring only what you write with: pens, pencils, erasers, . . . (You will be supplied with scratch paper.) You do NOT need a calculator for this exam.
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages.
- There are a total of **100** points.

Problem 1. (14%) Every year at the county fair a prize is given to the heaviest pig. Suppose n pigs are entered in the competition and the pigs are weighed in a random order. If the i^{th} pig weighed is heavier than the previous $i - 1$ pigs, what is the probability it will receive the prize? (Assume there are no ties.)

Problem 2. (14%) Let $X \sim \text{Poisson}(1)$. Calculate $E\left(\frac{1}{(X+1)(X+2)}\right)$.

Problem 3. (16%) Find the density of $Y = \sin^2(\pi X)$ where X is a random variable with density $f_X(x) = 3x^2$ for $0 < x < 1$.

Note: $\frac{d}{du} \sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}$

Problem 4. A monkey is tossing a biased coin which has probability of heads $1/3$ on each toss. The monkey records each toss of the coin but makes many errors, recording a head as a tail with probability $1/5$, and recording a tail as a head with probability $1/4$. Assume the tosses are independent.

(a) (8%) If the monkey tosses the coin once and records a head, what is the probability he actually tossed a head?

(b) (8%) If the monkey tosses the coin twice and records two heads, what is the probability he actually tossed two heads? (Give a detailed argument.)

Problem 5. (14%) A monkey types **5** letters at random. (Each keystroke is independent of the others with all 26 possibilities equally likely.)

What is the probability the monkey types the word ALL? (That is, the letters ALL occur as three consecutive letters somewhere in the **5** typed letters.)

Problem 6. (14%) Suppose A and B are mutually exclusive events. State and prove a simple expression for $P(A \mid A \cup B)$.

No work is required for the problems on this page. Just state or circle the correct answer.

Problem 7. A hat contains 3 coins, numbered 1, 2, 3. A simple experiment consists of choosing a coin from the hat and tossing it four times. (The same coin is tossed four times.)

(a) (3%) Describe the sample space Ω of this experiment, and give an example of a particular outcome ω in this sample space.

(b) (3%) What is $\#(\Omega)$, the number of possible outcomes in the experiment?

Problem 8. (3%) Which one of the following is a correct expression for $P(\text{at most one of } A \text{ or } B)$? (Circle the single correct answer.)

- a) $1 - P(A) - P(B)$ b) $P(A) + P(B) - P(A \cap B)$ c) $P(A) + P(B) - 2P(A \cap B)$
d) $1 - P(A \cap B)$ e) $P(A) + P(B)$ f) $P(A)P(B)$ g) $P(A) + P(B) - P(A)P(B)$

Problem 9. (3%) Suppose we define

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } 0 < x < 0.3 \\ \boxed{} & \text{for } 0.3 \leq x \leq 0.7 \\ x & \text{for } 0.7 < x < 1 \\ 1 & \text{for } x \geq 1 \end{cases}$$

There is an empty box in the definition of F . Circle the **single** expression from the list below which makes F into a cdf when it is placed in the box.

- a) 0.2 b) 0.3 c) 0.5 d) 0.7 e) 0.8
f) $0.5x$ g) $0.5x + 0.15$ h) $0.5x + 0.3$ i) $0.5x + 0.4$