TEST #1
STA 5326 **Name:**_____
September 24, 2015

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**. (You will have access to a copy of the "Table of Common Distributions" given in the back of the text.)
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- **If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.**
- Show and explain your work (including your calculations) for all problems except those on the last page. **No credit is given without work.** But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- All the work on the exam should be your own. No "cooperation" is allowed.
- Simplify your answers when it is easy to do so. But more difficult arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate large binomial coefficients, factorials or powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- You need to bring only what you write with: pens, pencils, erasers, ... (You will be supplied with scratch paper.) You do NOT need a calculator for this exam.
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages.
- There are a total of **100** points.

General Remark: I do NOT require students to simplify their arithmetic. They can leave in fractions, powers, factorials, etc. If you can see that their answer is correct, give them full credit. But students **must** *do all the necessary calculus to get full credit. They must compute all derivatives; they should lose points if they leave 'd/dx's or 'primes' in their answer. Also, student must simplify summations if there is a simple closed form.*

Problem 1. (14%) Every year at the county fair a prize is given to the heaviest pig. Suppose n pigs are entered in the competition and the pigs are weighed in a random order. If the i^{th} pig weighed is heavier than the previous $i-1$ pigs, what is the probability it will receive the prize? (Assume there are no ties.)

This is the same as problem 1.32, but with a different story. There are two different solutions given in pages 17-20 of solutions1_text.pdf.

Students might remember the answer, which is pretty simple, but they should get **no** *credit for stating the answer without any work.*

Problem 2. (14%) Let $X \sim \text{Poisson}(1)$. Calculate $E\left(\dfrac{1}{(X+1)(X+2)}\right)$.

This is similar to Exercise A1(b). The solution is also very similar.
The answer is $1 - P(X=0) - P(X=1) = 1 - 1/e - 1/e = 1 - 2/e$.

Students should simplify the answer to receive full credit; they should **not** *leave their answer as a summation.*
They can receive partial credit for correctly writing the answer as a summation, but if they leave it at that they should receive no more than half the credit.

Problem 3. (16%) Find the density of $Y = \sin^2(\pi X)$ where X is a random variable with density $f_X(x) = 3x^2$ for $0 < x < 1$.

Note: $\dfrac{d}{du}\sin^{-1}(u) = \dfrac{1}{\sqrt{1-u^2}}$

This problem is very similar to a homework problem and a lecture example. See Exercise 2.5 and notes3.pdf, pages 22-25.

Problem 4. A monkey is tossing a biased coin which has probability of heads 1/3 on each toss. The monkey records each toss of the coin but makes many errors, recording a head as a tail with probability 1/5, and recording a tail as a head with probability 1/4. Assume the tosses are independent.

This is problem 1.41 with a new story and different numbers.

(a) (8%) If the monkey tosses the coin once and records a head, what is the probability he actually tossed a head?

(b) (8%) If the monkey tosses the coin twice and records two heads, what is the probability he actually tossed two heads? (Give a detailed argument.)

The solution manual solution for 1.41(b) is not sufficient. The answer to part (b) of this problem is just the square of the answer to part (a), but some argument for why this is true should be given. See page 15 of solutions1_text.pdf for a good argument.

If students just state that the answer to part (b) is the square of the answer to part (a), but don't give a sufficient argument for this, they should get no more than 5 out of the 8 points.

Problem 5. (14%) A monkey types **5** letters at random. (Each keystroke is independent of the others with all 26 possibilities equally likely.)

What is the probability the monkey types the word ALL? (That is, the letters ALL occur as three consecutive letters somewhere in the **5** typed letters.)

The solution to this is the same as that for P(monkey types ZIT) on page 12 of notes1.pdf.

Problem 6. (14%) Suppose A and B are mutually exclusive events. State and prove a simple expression for $P(A \mid A \cup B)$.

This is the same as exercise 1.38(c). The solution manual solution on page 9 of solutions1_text.pdf is fine and gives sufficient detail.
The answer is $P(A \mid A \cup B) = \dfrac{P(A)}{P(A) + P(B)}$.

The answer $\dfrac{P(A)}{P(A) + P(B) - P(A \cap B)}$ *is technically correct, but should* **NOT** *get full credit since it is not as simple as the best answer and because it makes no use of the fact that* A *and* B *are mutually exclusive (disjoint).*

The answer $\dfrac{P(A)}{P(A) + P(B) - P(A)P(B)}$ *is wrong and should lose even more credit than the above. This answer indicates the student is confused between "mutually exclusive" and "independence".*

No work is required for the problems on this page. Just state or circle the correct answer.

Problem 7. A hat contains 3 coins, numbered 1, 2, 3. A simple experiment consists of choosing a coin from the hat and tossing it four times. (The same coin is tossed four times.)

This is similar to the example on page 28 of notes1.pdf.
You can give partial credit in the parts of this problem, if you see their description or counting is almost right. But if they make a major error, it is best to just give them a zero for that part.

(a) (3%) Describe the sample space Ω of this experiment, and give an example of a particular outcome ω in this sample space.

(b) (3%) What is $\#(\Omega)$, the number of possible outcomes in the experiment?

Problem 8. (3%) Which one of the following is a correct expression for $P(\text{at } \textbf{most} \text{ one of } A \text{ or } B)$? (Circle the single correct answer.)

a) $1 - P(A) - P(B)$ **b**) $P(A) + P(B) - P(A \cap B)$ **c**) $P(A) + P(B) - 2P(A \cap B)$
d)$\star$ $1 - P(A \cap B)$ **e**) $P(A) + P(B)$ **f**) $P(A)P(B)$ **g**) $P(A) + P(B) - P(A)P(B)$

This is Exercise 1.4(d).

Problem 9. (3%) Suppose we define

$$F(x) = \begin{cases} 0 & \text{for } x \le 0 \\ x & \text{for } 0 < x < 0.3 \\ \boxed{} & \text{for } 0.3 \le x \le 0.7 \\ x & \text{for } 0.7 < x < 1 \\ 1 & \text{for } x \ge 1 \end{cases}$$

There is an empty box in the definition of F. Circle the **single** expression from the list below which makes F into a cdf when it is placed in the box.

a) 0.2 **b**) 0.3 **c**) 0.5 **d**)$\star$ 0.7 **e**) 0.8
f) $0.5x$ **g**) $0.5x + 0.15$ **h**) $0.5x + 0.3$ **i**) $0.5x + 0.4$

The answer 0.7 is the only one that makes F **right continuous** *in addition to nondecreasing.*