TEST #2
STA 5326 **Name:**______________________________________
October 29, 2015

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- You will have access to a copy of the "Table of Common Distributions" given in the back of the text and to a table of the normal distribution.
- For problems requiring probability approximations or the use of tables, you must give a numerical answer in order to receive full credit.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- Show and explain your work (including your calculations) for all problems except those on the last two pages. **No credit is given without work.** But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- All the work on the exam should be your own. No "cooperation" is allowed.
- You need to bring only what you write with: pens, pencils, erasers, ... (You will be supplied with scratch paper.) A calculator would be somewhat helpful, but is not absolutely essential. (The arithmetic is not too bad.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **8** pages.
- There are a total of **100** points.

Problem 1. (16%) Suppose λ and c are arbitrary real numbers. Let $p_1, p_2, p_3, \dots$ be any sequence of values such that $p_n \to 1$ and $n(1 - p_n) \to \lambda$ as $n \to \infty$. Evaluate the following limit. Show your work.

$$\lim_{n\to\infty} \left(\frac{p_n}{1 - (1 - p_n)c} \right)^n$$

Problem 2. (16%) Suppose X has density $f_X(x) = \frac{1}{\beta}e^{-x/\beta}$ for $x \geq 0$. Find the variance of $Y = X^\delta$ for $\delta > 0$.
(Show your work. **Do NOT use results from the appendix** in your solution to this problem.)

Problem 3. A standard drug is known to be effective in 75% of the cases in which it is used. A less expensive new drug is currently being tested.

(a) (14%) The new drug is tested on 432 patients and found to be effective in only 301 cases. Is the new drug **inferior** to the old one? (Answer this question by computing an approximation to a relevant probability and then drawing a conclusion from this.)

[**Problem 3 continued**]

(b) (12%) Suppose now that each drug is tested on 600 patients (so that there are 1200 patients in total). If the two drugs are actually equally effective, what is the approximate probability that the old drug will be effective on **at least 23 more** patients than the new drug?

Problem 4. (15%) A carnival game has a bucket of n hollow balls, each containing a prize. One of the prizes is an expensive ring; the others are cheap trinkets. If you pay a dollar, you get to draw a ball from the bucket and keep the prize. A man decides to keep playing (and keep paying a dollar per ball) until he wins the ring. **The bucket is only refilled with balls after each player is completely finished playing.** Let X be the amount of money the man ends up paying in his quest for the ring. Find EX.

The remaining problems require no work. You will receive full credit just for stating the correct answer. However, some work space is provided in case needed. In the multiple choice problems, circle the single correct response **except in one problem** which is clearly identified.

Problem 5. (5%) Let T_r denote the time of the r^{th} arrival in a Poisson process with constant rate λ. Express $P(T_r > t)$ as an explicit summation of simple terms. (Hint: Use a relationship between T_r and S_t, the number of arrivals by time t.)

Problem 6. (3%) Suppose the random variable X has density $f_X(x) = 5e^{-5x}$ for $x \geq 0$. What is the hazard function of X?

Problem 7. (3%) Suppose X is a random variable with density $f_X(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$, which is a special case of the double exponential distribution. Let $M_X(t)$ denote the moment generating function (mgf) of X. What is the value of $M_X(2)$?

a) $-1/3$ **b)** $1/3$ **c)** -1 **d)** 1 **e)** $-\frac{e^2}{3}$

f) $\frac{e^2}{3}$ **g)** $-\frac{e}{3}$ **h)** $\frac{e}{3}$ **i)** ∞ **j)** 0

Problem 8. (3%) Suppose X is a random variable with density $h(x)$ which satisfies $h(x) > 0$ for $x \geq 1$ and $h(x) = 0$ for $x < 1$. Let $\cdot$ denote multiplication. The integral given below

$$\int_0^\infty t \cdot h\left(e^{\sqrt{t}}\right) \cdot \left(\frac{d}{dt} e^{\sqrt{t}}\right) \, dt$$

is equal to ...

a) $E\left[\frac{\sqrt{t}}{2} e^{\sqrt{X}}\right]$ **b)** $E\left[(\log X)^2\right]$ **c)** $E\left[e^{\sqrt{X}}\right]$ **d)** $E\left[h(e^{\sqrt{X}})\right]$

e) mgf of $\sqrt{X}$ **f)** mgf of $\log X$ **g)** mgf of $e^{\sqrt{X}}$ **h)** mgf of $\frac{\sqrt{t}}{2} e^{\sqrt{X}}$

In the next two problems, suppose $X_1, X_2, X_3, \ldots$ are iid with mean μ and variance $\sigma^2 < \infty$. Let $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$.

Problem 9. (4%) Circle **ALL** of the true statements below.

a) $P\left(\lim_{n\to\infty} \bar{X}_n = \mu\right) = 1$ **b)** $\lim_{n\to\infty} P\left(|\bar{X}_n - \mu| > \varepsilon\right) = 1$ for all $\varepsilon > 0$

c) $\lim_{n\to\infty} P\left(\bar{X}_n = \mu\right) = 1$ **d)** $\lim_{n\to\infty} P\left(|\bar{X}_n - \mu| < \varepsilon\right) = 1$ for all $\varepsilon > 0$

e) $\lim_{n\to\infty} P\left(|\bar{X}_n - \mu| = 0\right) = 1$ **f)** $P\left(\lim_{n\to\infty} |\bar{X}_n - \mu| > \varepsilon\right) = 1$ for all $\varepsilon > 0$

Problem 10. (3%) Which ONE of the following statements is true? Circle the single correct response.

a) $P\left(|\bar{X}_n - \mu| > \varepsilon\right) > \frac{\sigma^2}{n\varepsilon^2}$ **b)** $P\left(|\bar{X}_n - \mu| > \varepsilon\right) < \frac{\sigma^2}{n\varepsilon^2}$ **c)** $P\left(|S_n - n\mu| > \varepsilon\right) < \frac{\sigma^2}{n\varepsilon^2}$

d) $P\left(|\bar{X}_n - \mu| > \varepsilon\right) > \frac{n\sigma^2}{\varepsilon^2}$ **e)** $P\left(|S_n - n\mu| > \varepsilon\right) < \frac{n\varepsilon^2}{\sigma^2}$ **f)** $P\left(|S_n - n\mu| > \varepsilon\right) > \frac{n\varepsilon^2}{\sigma^2}$

Problem 11. (3%) Consider a distribution with density

$$f(x) = \frac{1}{10} e^{-|x-3|/5}, \quad -\infty < x < \infty.$$

What is the **median** of this distribution?

a) 3 **b)** 5 **c)** 9 **d)** 10 **e)** 25 **f)** Does NOT exist

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Problem 12. (3%) Consider a distribution with density

$$f(x) = \frac{1}{2} \frac{1}{(1 + |x-3|)^2}, \quad -\infty < x < \infty.$$

What is the **mean** of this distribution?

a) 0 **b)** 2 **c)** 3 **d)** 4 **e)** 9 **f)** Does NOT exist