TEST #3
STA 5326 **Name:**______________________________________
December 3, 2015

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- You will have access to a copy of the "Table of Common Distributions" given in the back of the text.
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- Show and explain your work (including your calculations) for all problems (except those on the last page). **No credit is given without work.** But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- All the work on the exam should be your own. No "cooperation" is allowed.
- You need to bring only what you write with: pens, pencils, erasers, . . . (You will be supplied with scratch paper.) A calculator is NOT needed for this exam.
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- The exam has **9** pages.
- There are a total of **100** points.

Problem 1. (12%) Prove that if the joint cdf of X and Y satisfies

$$F_{X,Y}(x,y) = F_X(x)F_Y(y) \quad \text{for all } x, y,$$

then for any values a and b,

$$P(X > a, Y > b) = P(X > a)P(Y > b)\,.$$

Explain your argument.

Problem 2. (14%) Let X, Y, Z be independent random variables with means μ_X, μ_Y, μ_Z and variances $\sigma_X^2, \sigma_Y^2, \sigma_Z^2$. Find an expression for the correlation between $XY + 5$ and $YZ - 6$ in terms of these means and variances.

Problem 3. Suppose $X \mid P \sim \text{Negative Binomial}(r, P)$ and $P \sim \text{Beta}(\alpha, \beta)$.

(a) (14%) Find EX.

[**Problem 3 continued**]

(b) (14%) Find the marginal pmf of X.

Problem 4. Suppose the joint pdf of (X, Y) is

$$f(x, y) = 24y(x - y) \quad \text{for } 0 < y < x < 1.$$

(The support of this joint pdf is a triangular region bounded by the lines $y = 0$, $x = 1$, and $y = x$.)

(a) (10%) Calculate $F_{X,Y}(1/2, 2)$, the value of the joint cdf at the point $(1/2, 2)$.

[**Problem 4 continued**]

(b) (16%) Let

$$U = \frac{1}{X} \quad \text{and} \quad V = \frac{Y}{X}\,.$$

Find the joint pdf of (U, V). (Don't forget to specify the support of the joint pdf.)

[**Problem 4 continued**]

(c) (10%) Find the marginal pdf of V. (Don't forget to specify the support.)

You are NOT required to show any work for the problems on this page. You will receive full credit just for filling in the blanks correctly. Some space is provided in case you should need it.

Problem 5. (4%) Suppose X, Y are iid with common density $f(x) = 2x$ for $0 < x < 1$ (and $f(x) = 0$ otherwise). The pdf of $Z = X + Y$ can be computed as

$$f_Z(z) = \begin{cases} \displaystyle\int_A^B 4x(z-x)\,dx & \text{for } 0 < z \le 1, \\ \displaystyle\int_C^D 4x(z-x)\,dx & \text{for } 1 < z \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

What are the correct values for A, B, C, D that appear above? (Fill in the blanks.)

A=____________ B=____________ C=____________ D=____________

Problem 6. (6%) Suppose $X \sim N(2, 3)$ and $Y \mid X \sim N(X, 5)$. What is the marginal distribution of Y? (Fill in the blanks below. The first blank should contain the name of a distribution, and the second and third blanks should contain numbers.)

The marginal for Y has a

________________ distribution with mean = ________ and variance = ________.