NWR = No Work Required **For parts that are labeled NWR, you will get full credit just for stating the correct answer. You do NOT have to show any work or give any explanation.**

Please read the following directions. **(They are included in the directions I posted to the Canvas Announcements, so you can skip the following if you have already read those directions.)**

- The exam is closed book and closed notes. No books, notes, or internet resources are allowed.
- A copy of the Table of Common Distributions from the back of our textbook is attached to the end of your exam.
- During the exam, you need only a supply of blank paper and writing implements (pens, pencils, erasers, etc). If you wish, you may use an ordinary scientific calculator (TI-86 or below is fine).
- Partial credit is available. (If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.)
- You must show and explain your work (including your calculations). **No credit is given without work or explanation!** But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, this work should be copied to your written solutions.
- **If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.**
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- Simplify your answers when it is easy to do so. But more difficult arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate large binomial coefficients, factorials or powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- All algebra and calculus must be done completely. (Only arithmetic can be left incomplete.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- All the work on the exam should be your own. No "cooperation" is allowed.
- The exam has **6** problems and there are a total of **100** points.

Problem 1. Suppose X has density

$$f(x) = \begin{cases} \frac{2}{\sqrt{\pi}} e^{-x^2} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0\,. \end{cases}$$

(a) (8%) Find EX.

(b) (8%) Find $\text{Var}(X)$.

Problem 2. (10%) Suppose $X \sim$ Negative Binomial(r, p) and $Y \sim$ Binomial(n, p). (Here we use the lecture definition of the Negative Binomial distribution in which Y is the number of trials needed to get r successes. This is referred to as the "alternative form" in the appendix.)

Show that $F_X(n) = 1 - F_Y(r-1)$.

Problem 3. A *truncated* discrete distribution is one in which a particular class cannot be observed and is eliminated from the sample space. In particular, if X has range 0, 1, 2, ... and the 0 class cannot be observed, the 0-*truncated* random variable X_T has mass function (pmf)

$$P(X_T = x) = \frac{P(X = x)}{P(X > 0)}, \quad x = 1, 2, 3, \ldots$$

Let X_T be the 0-*truncated* random variable X_T obtained starting from $X \sim$ Binomial(n, p).

(a) (8%) Find the mean of X_T.

(b) (8%) Find the variance of X_T.

Problem 4. Let X be a random variable with moment generating function (mgf) given by

$$M(t) = 1 + \frac{pt}{1-t} \quad \text{for } t < 1\,.$$

where p is some value in the range $0 \leq p \leq 1$. (Any value in this range leads to a valid mgf.)

(a) (8%) Find EX.

(b) (8%) Find $\text{Var}(X)$.

(c) (6%, **NWR**) Let $X_1, X_2, \ldots, X_n$ be iid with the mgf $M(t)$ given above. Define

$$Y = \sum_{i=1}^{n} X_i\,.$$

Find $M_Y(t)$, the mgf of Y.

(d) (8%) Let $\beta > 0$. Find the limit of $M_Y(t)$ as $n \to \infty$, $p \to 0$, and $np \to \beta$. (For example, you could take $p_n = \beta/n$ for all n.)

Problem 5. There are **four** snipers firing at the enemy from hidden positions. The enemy is constantly searching for the snipers' locations, and when a sniper's location is discovered, she is killed in a hail of bullets. Assume that the snipers begin their work at the same time and that the snipers' "lifetimes" (the time until they are discovered and killed) are i.i.d. exponential random variables with a mean of β hours.

(a) (4%, **NWR**) Find the mean of the time until the last sniper dies.

(b) (4%, **NWR**) Find the variance of the time until the last sniper dies.

(c) (8%) The snipers are killed off one by one. Assume that, while k snipers remain alive, they kill the enemy soldiers at an average rate of ck^2 deaths per hour. (c is an arbitrary positive value.) What is the expected value of the total number of enemy soldiers killed by the four snipers?

Problem 6. Let X have density

$$f_X(x) = \begin{cases} \dfrac{\cos(x)}{B\sqrt{1-x^2}} & \text{for } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

where $B \approx 2.403939431$.

(a) (6%) Does EX^{13} exist? (In other words, is EX^{13} a well-defined finite value?) Answer 'YES' or 'NO' and state a reason to justify your answer.

(b) (6%) Let $M_X(t)$ be the mgf of X. For what values of t is $M_X(t)$ finite? State a reason to justify your answer.

Table of Common Distributions

Do Not Mark !!

Discrete Distributions

Bernoulli(p)

pmf	$P(X = x\|p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = p, \quad \mathrm{Var}\, X = p(1-p)$
mgf	$M_X(t) = (1-p) + pe^t$

Binomial(n, p)

pmf	$P(X = x\|n, p) = \binom{n}{x} p^x(1-p)^{n-x}; \quad x = 0, 1, 2, \ldots, n; \quad 0 \le p \le 1$
mean and variance	$\mathrm{E}X = np, \quad \mathrm{Var}\, X = np(1-p)$
mgf	$M_X(t) = [pe^t + (1-p)]^n$

notes Related to Binomial Theorem (Theorem 3.1.1). The *multinomial* distribution (Definition 4.6.1) is a multivariate version of the binomial distribution.

Discrete Uniform

pmf	$P(X = x\|N) = \frac{1}{N}; \quad x = 1, 2, \ldots, N; \quad N = 1, 2, \ldots$
mean and variance	$\mathrm{E}X = \frac{N+1}{2}, \quad \mathrm{Var}\, X = \frac{(N+1)(N-1)}{12}$
mgf	$M_X(t) = \frac{1}{N}\sum_{i=1}^{N} e^{it}$

Geometric(p)

pmf $P(X = x|p) = p(1-p)^{x-1}; \quad x = 1, 2, \ldots; \quad 0 \le p \le 1$

mean and variance $\mathrm{E}X = \frac{1}{p}, \quad \mathrm{Var}\, X = \frac{1-p}{p^2}$

mgf $M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\log(1-p)$

notes $Y = X - 1$ is negative binomial$(1, p)$. The distribution is *memoryless*: $P(X > s|X > t) = P(X > s - t)$.

Hypergeometric

pmf $P(X = x|N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; \quad x = 0, 1, 2, \ldots, K;$
$M - (N - K) \le x \le M; \quad N, M, K \ge 0$

mean and variance $\mathrm{E}X = \frac{KM}{N}, \quad \mathrm{Var}\, X = \frac{KM}{N}\frac{(N-M)(N-K)}{N(N-1)}$

notes If $K << M$ and N, the range $x = 0, 1, 2, \ldots, K$ will be appropriate.

Negative binomial(r, p)

pmf $P(X = x|r, p) = \binom{r+x-1}{x} p^r (1-p)^x; \quad x = 0, 1, \cdots; \quad 0 \le p \le 1$

mean and variance $\mathrm{E}X = \frac{r(1-p)}{p}, \quad \mathrm{Var}\, X = \frac{r(1-p)}{p^2}$

mgf $M_X(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r, \quad t < -\log(1-p)$

notes An alternate form of the pmf is given by $P(Y = y|r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, $y = r, r+1, \ldots$. The random variable $Y = X + r$. The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.34.)

Poisson(λ)

pmf $P(X = x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \le \lambda < \infty$

mean and variance $\mathrm{E}X = \lambda, \quad \mathrm{Var}\, X = \lambda$

mgf $M_X(t) = e^{\lambda(e^t - 1)}$

Continuous Distributions

Beta(α, β)

pdf $f(x|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1, \quad \alpha > 0, \quad \beta > 0$

mean and variance $\mathrm{E}X = \frac{\alpha}{\alpha+\beta}, \quad \mathrm{Var}\, X = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

mgf $M_X(t) = 1 + \sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$

notes The constant in the beta pdf can be defined in terms of gamma functions, $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. Equation (3.2.18) gives a general expression for the moments.

Cauchy(θ, σ)

pdf $f(x|\theta,\sigma) = \frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty, \quad \sigma > 0$

mean and variance do not exist

mgf does not exist

notes Special case of Student's t, when degrees of freedom $= 1$. Also, if X and Y are independent n(0, 1), X/Y is Cauchy.

Chi squared

pdf $f(x|p) = \frac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}; \quad 0 \le x < \infty; \quad p = 1, 2, \ldots$

mean and variance $\mathrm{E}X = p, \quad \mathrm{Var}\, X = 2p$

mgf $M_X(t) = \left(\frac{1}{1-2t}\right)^{p/2}, \quad t < \frac{1}{2}$

notes Special case of the gamma distribution.

Double exponential(μ, σ)

pdf $f(x|\mu,\sigma) = \frac{1}{2\sigma} e^{-|x-\mu|/\sigma}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

mean and variance $\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = 2\sigma^2$

mgf $M_X(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}, \quad |t| < \frac{1}{\sigma}$

notes Also known as the *Laplace* distribution.

Exponential(β)

pdf $f(x|\beta) = \frac{1}{\beta} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \beta > 0$

mean and variance $\mathrm{E}X = \beta, \quad \mathrm{Var}\, X = \beta^2$

mgf $M_X(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$

notes Special case of the gamma distribution. Has the *memoryless* property. Has many special cases: $Y = X^{1/\gamma}$ is *Weibull*, $Y = \sqrt{2X/\beta}$ is *Rayleigh*, $Y = \alpha - \gamma \log(X/\beta)$ is *Gumbel*.

F

pdf $f(x|\nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}}; \quad 0 \le x < \infty;$
$\nu_1, \nu_2 = 1, \ldots$

mean and variance $\mathrm{E}X = \frac{\nu_2}{\nu_2-2}, \quad \nu_2 > 2,$

$\mathrm{Var}\, X = 2\left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}, \quad \nu_2 > 4$

moments (mgf does not exist) $\mathrm{E}X^n = \frac{\Gamma\left(\frac{\nu_1+2n}{2}\right)\Gamma\left(\frac{\nu_2-2n}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_2}{\nu_1}\right)^n, \quad n < \frac{\nu_2}{2}$

notes Related to chi squared ($F_{\nu_1,\nu_2} = \left(\frac{\chi^2_{\nu_1}}{\nu_1}\right) \Big/ \left(\frac{\chi^2_{\nu_2}}{\nu_2}\right)$, where the χ^2s are independent) and t ($F_{1,\nu} = t^2_\nu$).

Gamma(α, β)

pdf $f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha,\ \beta > 0$

mean and variance $\mathrm{E}X = \alpha\beta, \quad \mathrm{Var}\, X = \alpha\beta^2$

mgf $M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha, \quad t < \frac{1}{\beta}$

notes Some special cases are exponential ($\alpha = 1$) and chi squared ($\alpha = p/2, \beta = 2$). If $\alpha = \frac{3}{2}$, $Y = \sqrt{X/\beta}$ is *Maxwell*. $Y = 1/X$ has the *inverted gamma distribution*. Can also be related to the Poisson (Example 3.2.1).

Logistic(μ, β)

pdf $f(x|\mu,\beta) = \frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty,$ $\beta > 0$

mean and variance $EX = \mu, \quad \text{Var}\, X = \frac{\pi^2\beta^2}{3}$

mgf $M_X(t) = e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t), \quad |t| < \frac{1}{\beta}$

notes The cdf is given by $F(x|\mu,\beta) = \frac{1}{1+e^{-(x-\mu)/\beta}}$.

Lognormal(μ, σ^2)

pdf $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty,$ $\sigma > 0$

mean and variance $EX = e^{\mu+(\sigma^2/2)}, \quad \text{Var}\, X = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

moments (mgf does not exist) $EX^n = e^{n\mu+n^2\sigma^2/2}$

notes Example 2.3.5 gives another distribution with the same moments.

Normal(μ, σ^2)

pdf $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty,$ $-\infty < \mu < \infty, \quad \sigma > 0$

mean and variance $EX = \mu, \text{Var}\, X = \sigma^2$

mgf $M_X(t) = e^{\mu t+\sigma^2 t^2/2}$

notes Sometimes called the *Gaussian* distribution.

Pareto(α, β)

pdf $f(x|\alpha,\beta) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \alpha < x < \infty, \quad \alpha > 0, \quad \beta > 0$

mean and variance $EX = \frac{\beta\alpha}{\beta-1}, \quad \beta > 1,$

$\text{Var}\, X = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \quad \beta > 2$

mgf does not exist

t

pdf	$f(x\|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{(\nu+1)/2}}, \quad -\infty < x < \infty, \quad \nu = 1, \ldots$
mean and variance	$\mathrm{E}X = 0, \quad \nu > 1,$ $\mathrm{Var}\, X = \frac{\nu}{\nu-2}, \quad \nu > 2$
moments (mgf does not exist)	$\mathrm{E}X^n = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{\nu-n}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)} \nu^{n/2}$ if $n < \nu$ and even, $\mathrm{E}X^n = 0$ if $n < \nu$ and odd.
notes	Related to F ($F_{1,\nu} = t_\nu^2$).

Uniform(a, b)

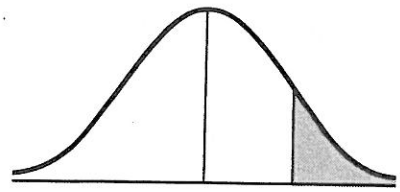
pdf	$f(x\|a, b) = \frac{1}{b-a}, \quad a \le x \le b$
mean and variance	$\mathrm{E}X = \frac{b+a}{2}, \quad \mathrm{Var}\, X = \frac{(b-a)^2}{12}$
mgf	$M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$
notes	If $a = 0$ and $b = 1$, this is a special case of the beta ($\alpha = \beta = 1$).

Weibull(γ, β)

pdf	$f(x\|\gamma, \beta) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x^\gamma/\beta}, \quad 0 \le x < \infty, \quad \gamma > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \beta^{1/\gamma}\Gamma\left(1 + \frac{1}{\gamma}\right), \quad \mathrm{Var}\, X = \beta^{2/\gamma}\left[\Gamma\left(1 + \frac{2}{\gamma}\right) - \Gamma^2\left(1 + \frac{1}{\gamma}\right)\right]$
moments	$\mathrm{E}X^n = \beta^{n/\gamma}(1 + \frac{n}{\gamma})$

notes The mgf exists only for $\gamma \ge 1$. Its form is not very useful. A special case is exponential ($\gamma = 1$).

TABLE 1 Normal distribution, right-hand tail probabilities



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8557	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1,6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998